→ <u>shrnutí</u>

kvadrant	sin	cos	tg	cotg
I → x	+	+	+	+
II → $\pi - x$	+	−	−	−
III → $\pi + x$	−	−	+	+
IV → $2\pi - x$	−	+	−	−
perioda	2π	2π	π	π
parita	lichá	sudá	lichá	lichá

	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$f(x)=0$
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	$x = k\pi$
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	$x = \frac{\pi}{2} + k\pi$
tg	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	−	0	$x = k\pi$
cotg	−	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0	−	$x = \frac{\pi}{2} + k\pi$

Goniometrické vzorce

Thursday, April 30, 2020 11:07 AM

Vzorce pro funkce o argumentu 2·x a $\frac{1}{2}\cdot x$

- $\sin(2\cdot x) = 2\cdot\sin(x)\cdot\cos(x)$
- $\cos(2\cdot x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1$
- $\mathrm{tg}(2\cdot x) = \frac{2\cdot\mathrm{tg}(x)}{1-\mathrm{tg}^2(x)}$ $= 1 - 2\sin^2(x)$

- $\left|\sin\left(\frac{x}{2}\right)\right| = \sqrt{\frac{1-\cos(x)}{2}}$
- $\left|\cos\left(\frac{x}{2}\right)\right| = \sqrt{\frac{1+\cos(x)}{2}}$
- $\left|\mathrm{tg}\left(\frac{x}{2}\right)\right| = \sqrt{\frac{1-\cos(x)}{1+\cos(x)}}$

Součtové vzorce

- $\sin(x + y) = \sin(x)\cdot\cos(y) + \cos(x)\cdot\sin(y)$
- $\sin(x - y) = \sin(x)\cdot\cos(y) - \cos(x)\cdot\sin(y)$

- $\cos(x + y) = \cos(x)\cdot\cos(y) - \sin(x)\cdot\sin(y)$
- $\cos(x - y) = \cos(x)\cdot\cos(y) + \sin(x)\cdot\sin(y)$

- $\mathrm{tg}(x + y) = \frac{\mathrm{tg}(x) + \mathrm{tg}(y)}{1 - \mathrm{tg}(x)\cdot\mathrm{tg}(y)}$
- $\mathrm{tg}(x - y) = \frac{\mathrm{tg}(x) - \mathrm{tg}(y)}{1 + \mathrm{tg}(x)\cdot\mathrm{tg}(y)}$

Vzorce na převod součtu na součin

- $\sin(x) + \sin(y) = 2\cdot\sin\left(\frac{x+y}{2}\right)\cdot\cos\left(\frac{x-y}{2}\right)$
- $\sin(x) - \sin(y) = 2\cdot\cos\left(\frac{x+y}{2}\right)\cdot\sin\left(\frac{x-y}{2}\right)$

- $\cos(x) + \cos(y) = 2\cdot\cos\left(\frac{x+y}{2}\right)\cdot\cos\left(\frac{x-y}{2}\right)$
- $\cos(x) - \cos(y) = -2\cdot\sin\left(\frac{x+y}{2}\right)\cdot\sin\left(\frac{x-y}{2}\right)$

- $\mathrm{tg}(x) + \mathrm{tg}(y) = \frac{\sin(x+y)}{\cos(x)\cdot\cos(y)}$
- $\mathrm{tg}(x) - \mathrm{tg}(y) = \frac{\sin(x-y)}{\cos(x)\cdot\cos(y)}$

Základní vzorce

- $\sin^2(x) + \cos^2(x) = 1$
- $\mathrm{tg}(x)\cdot\mathrm{cotg}(x) = 1 \quad \wedge \quad x \neq k\cdot\frac{\pi}{2}$

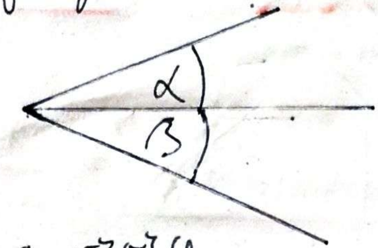
- $\sin(x) = \cos\left(\frac{\pi}{2} - x\right)$
- $\cos(x) = \sin\left(\frac{\pi}{2} - x\right)$
- $\mathrm{tg}(x) = \mathrm{cotg}\left(\frac{\pi}{2} - x\right)$
- $\mathrm{cotg}(x) = \mathrm{tg}\left(\frac{\pi}{2} - x\right)$

- $\mathrm{tg}(x) = \frac{\sin(x)}{\cos(x)} \quad \wedge \quad x \neq \frac{\pi}{2} + k\cdot\pi$
- $\mathrm{cotg}(x) = \frac{\cos(x)}{\sin(x)} \quad \wedge \quad x \neq k\cdot\pi$

- $\mathrm{tg}(x) = \frac{1}{\mathrm{cotg}(x)} \quad \wedge \quad x \neq k\cdot\frac{\pi}{2}$
- $\mathrm{cotg}(x) = \frac{1}{\mathrm{tg}(x)} \quad \wedge \quad x \neq k\cdot\frac{\pi}{2}$

Trigonometrie

- → užití goniometrických fcí pro řešení úloh s trojúhelníky
- → výškový úhel – α
- → hloubkový úhel – β



- → trigonometrické věty a vzorce
  - → pro každý $\triangle ABC$ s úhly α, β, γ a stranami a, b, c:
    - sinová věta

USU, SsU ← $$\frac{a}{\sin(\alpha)} = \frac{b}{\sin(\beta)} = \frac{c}{\sin(\gamma)} = 2r$$ ↗ poloměr kružnice opsané

- kosinová věta

SSS, SUS ← $$c^2 = a^2 + b^2 - 2ab\cdot\cos(\gamma)$$

- tangentová věta

$$\frac{a-b}{a+b} = \frac{tg\left(\frac{a-b}{2}\right)}{tg\left(\frac{a+b}{2}\right)} = \frac{tg\left(\frac{a-b}{2}\right)}{cotg\left(\frac{1}{2}\gamma\right)}$$

- vzorce pro obsah trojúhelníka
  - $S = \frac{a \cdot v_a}{2}$
  - $S = \frac{1}{2}ab\cdot\sin(\gamma) = \frac{1}{2}ac\cdot\sin(\beta) = \frac{1}{2}bc\cdot\sin(\alpha)$

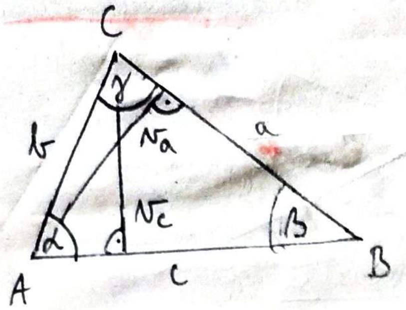
  → r = poloměr kružnice opsané

  → ρ = poloměr kružnice vepsané

  → $s = \frac{a+b+c}{2}$

  - $S = \rho \cdot s$
  - $S = \frac{a\cdot b\cdot c}{4r}$
  - $S = \sqrt{s(s-a)(s-b)(s-c)}$

→ odvození sinové věty

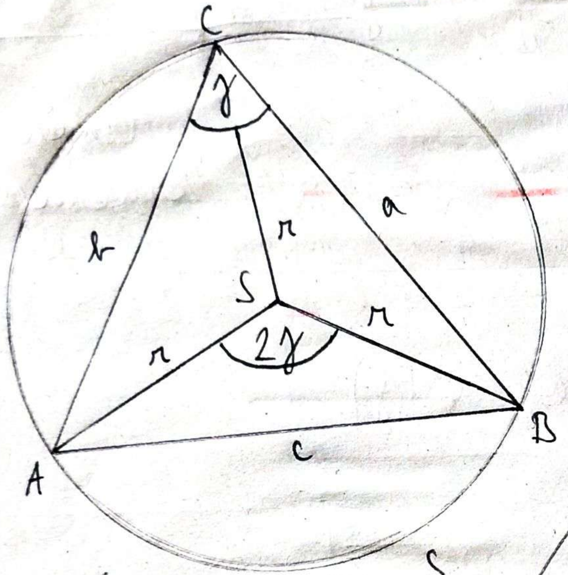


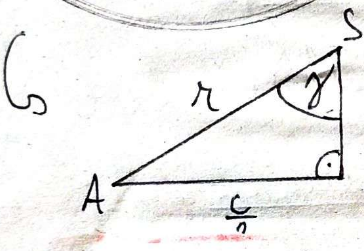
- $\sin(\beta) = \frac{v_c}{a} \Rightarrow v_c = a \cdot \sin(\beta)$
- $\sin(\alpha) = \frac{v_c}{b} \Rightarrow v_c = b \cdot \sin(\alpha)$

$$\frac{a}{\sin(\alpha)} = \frac{b}{\sin(\beta)}$$

- $\sin(\beta) = \frac{v_a}{c} \Rightarrow v_a = c \cdot \sin(\beta)$
- $\sin(\gamma) = \frac{v_a}{b} \Rightarrow v_a = b \cdot \sin(\gamma)$

$$\frac{c}{\sin(\gamma)} = \frac{b}{\sin(\beta)}$$

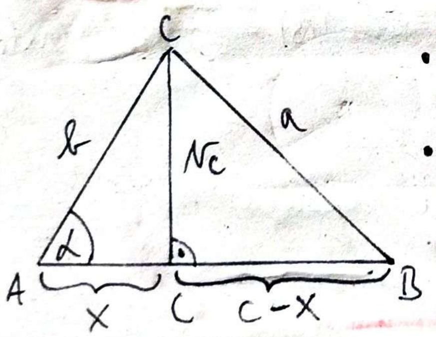




$$\sin(\gamma) = \frac{\frac{c}{2}}{r} \Rightarrow 2r = \frac{c}{\sin(\gamma)}$$

$$\Rightarrow \frac{a}{\sin(\alpha)} = \frac{b}{\sin(\beta)} = \frac{c}{\sin(\gamma)} = 2r$$

→ odvození kosinové věty



- $\sin(\alpha) = \frac{v_c}{b} \Rightarrow v_c = b \cdot \sin(\alpha)$
- $\cos(\alpha) = \frac{x}{b} \Rightarrow x = b \cdot \cos(\alpha)$

$$a^2 = v_c^2 + (c-x)^2$$

$$a^2 = b^2 \cdot \sin^2(\alpha) + (c - b\cos(\alpha))^2$$

$$a^2 = b^2 \cdot \sin^2(\alpha) + c^2 - 2bc \cdot \cos(\alpha) + b^2 \cdot \cos^2(\alpha)$$

$$a^2 = b^2(\sin^2(\alpha) + \cos^2(\alpha)) + c^2 - 2bc \cdot \cos(\alpha)$$

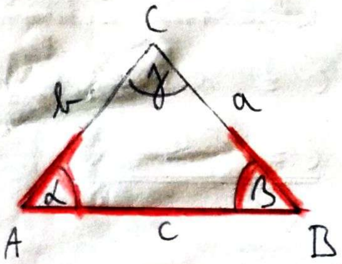
$$a^2 = b^2 + c^2 - 2bc \cdot \cos(\alpha)$$

cyklické záměny $\begin{cases} b^2 = a^2 + c^2 - 2ac \cdot \cos(\beta) \\ c^2 = a^2 + b^2 - 2ab \cdot \cos(\gamma) \end{cases}$

→ **příklady**

• $\triangle ABC: \alpha = 80°;\ \beta = 40°;\ c = 8 \rightarrow \gamma;\ a;\ b = ?$

USU



$\gamma = 60°$

$a = 9{,}1$

$b = 5{,}9$

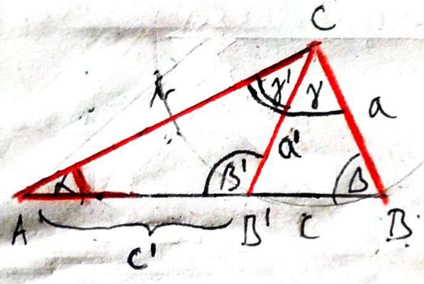
a) $\dfrac{a}{\sin(\alpha)} = \dfrac{c}{\sin(\gamma)}$

$a = \dfrac{8 \cdot \sin(80)}{\sin(60)} \doteq \underline{\underline{9{,}1}}$

b) $\dfrac{b}{\sin(\beta)} = \dfrac{c}{\sin(\gamma)}$

$b = \dfrac{8 \cdot \sin(40)}{\sin(60)} \doteq \underline{\underline{5{,}9}}$

• $\triangle ABC: a = 6;\ b = 7{,}7;\ \alpha = 50° \rightarrow c;\ \beta;\ \gamma = ?$



→ SsU ale mám úhel proti menší z nich

⇒ 2 řešení

$c = 6$

$\beta = 79{,}5°$

$\gamma = 50{,}5°$

$c' = 3{,}9$

$\beta' = 100{,}5°$

$\gamma' = 29{,}5°$

β) $\dfrac{\sin(\beta)}{b} = \dfrac{\sin(\alpha)}{a}$

$\sin(\beta) = \dfrac{b}{a} \cdot \sin(\alpha)$

$\beta = \arcsin\left(\dfrac{7{,}7}{6} \cdot \sin(50)\right)$

$\underline{\beta = 79{,}5°}$ → 1. kvadrant

$\beta' = 180 - \beta$

$\underline{\beta' \doteq 100{,}5°}$ → 2. kvadrant

γ) $\gamma = 180 - \alpha - \beta = \underline{\underline{50{,}5°}}$

$\gamma' = 180 - \alpha - \beta' = \underline{\underline{29{,}5°}}$

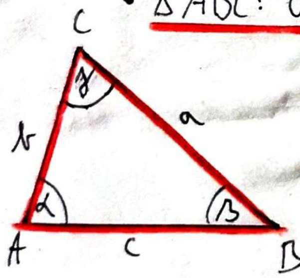
c) $\dfrac{c}{\sin(\gamma)} = \dfrac{a}{\sin(\alpha)}$

$c = \dfrac{a \cdot \sin(\gamma)}{\sin(\alpha)} = \dfrac{6 \cdot \sin(50{,}5)}{\sin(50)} \doteq \underline{\underline{6}}$

$c' = \dfrac{a \cdot \sin(\gamma')}{\sin(\alpha)} = \dfrac{a \cdot \sin(29{,}5)}{\sin(50)} \doteq \underline{\underline{3{,}9}}$

• $\triangle ABC: a = 15;\ b = 10;\ c = 12 \rightarrow \alpha, \beta, \gamma = ?$

SSS



$\alpha = 85°28'$

$\beta = 41°39'$

$\gamma = 52°53'$

α) $a^2 = b^2 + c^2 - 2bc \cdot \cos(\alpha)$

$2bc\cos(\alpha) = b^2 + c^2 - a^2$

$\cos(\alpha) = \dfrac{100 + 144 - 225}{2 \cdot 10 \cdot 12} = \dfrac{19}{240}$

$\alpha = \arccos\left(\dfrac{19}{240}\right) \doteq \underline{\underline{85{,}5°}}$

β) $\dfrac{\sin(\beta)}{b} = \dfrac{\sin(\alpha)}{a}$

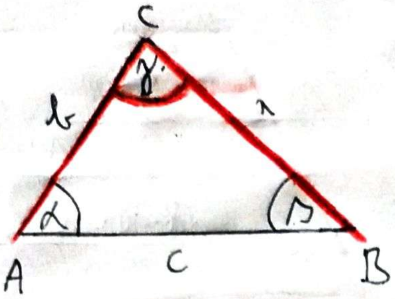
$\sin(\beta) = \dfrac{b \cdot \sin(\alpha)}{a}$

$\beta = \arcsin\left(\dfrac{2}{3} \cdot \sin(85{,}5)\right)$

$\underline{\underline{\beta \doteq 41{,}6°}}$

• $\triangle ABC$: $a = 15$ cm, $b = 13$ cm, $\gamma = 70° \rightarrow c, \alpha, \beta = ?$

SUS



$c = 16{,}14$ cm
$\alpha = 60°49'$
$\beta = 49°11'$

c) $c^2 = a^2 + b^2 - 2ab\cos(\gamma)$

$c^2 = 225 + 169 - 390 \cdot \cos(70)$

$\underline{\underline{c \doteq 16{,}14 \text{ cm}}}$

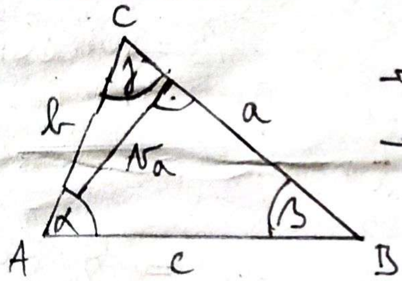
α) $\dfrac{\sin(\alpha)}{a} = \dfrac{\sin(\gamma)}{c}$

$\sin(\alpha) = \dfrac{15 \cdot \sin(70)}{16{,}14}$

$\underline{\underline{\alpha \doteq 60{,}8°}}$

→ odvození vzorců pro obsah trojúhelníku

• $S = \dfrac{a \cdot v_a}{2}$

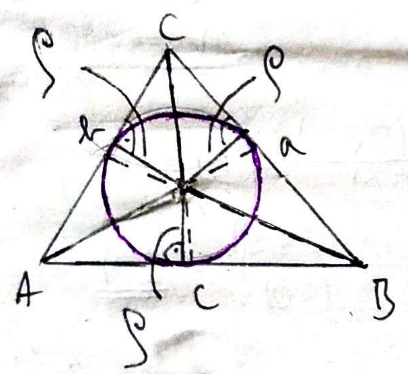


→ $v_a = c \cdot \sin(\beta)$ → $\underline{\underline{S = \frac{1}{2} a c \sin(\beta)}}$

→ $v_a = b \cdot \sin(\gamma)$ → $\underline{\underline{S = \frac{1}{2} ab \sin(\gamma)}}$

• $S = S_a + S_b + S_c = \frac{1}{2} a \cdot \varrho + \frac{1}{2} b \cdot \varrho + \frac{1}{2} c \cdot \varrho$

$= \varrho \cdot \frac{1}{2}(a + b + c) = \underline{\underline{\varrho \cdot \Delta}}$



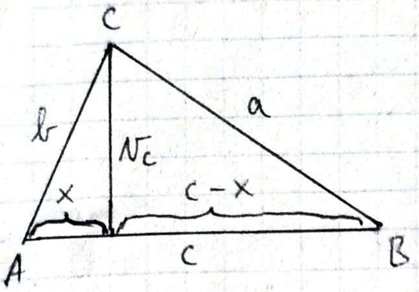
• $S = \frac{1}{2} ac \cdot \sin(\beta)$

$\dfrac{b}{\sin(\beta)} = 2r$

$\sin(\beta) = \dfrac{b}{2r}$

$S = \dfrac{a \cdot b \cdot c}{4r}$

• Heronův vzorec → viz papír



$$S = \frac{c \cdot h_c}{2} \rightarrow h_c = ?$$

$$\left.\begin{aligned} a^2 &= h_c^2 + (c-x)^2 \\ b^2 &= h_c^2 + x^2 \end{aligned}\right\} \underline{a^2 - (c-x)^2 = b^2 - x^2}$$

$$a^2 - c^2 + 2cx - x^2 = b^2 - x^2$$

$$x = \frac{b^2 - a^2 + c^2}{2c}$$

$$\Rightarrow \underline{h_c^2 = b^2 - \left(\frac{b^2 - a^2 + c^2}{2c}\right)^2} =$$

$$= \frac{4b^2c^2 - (b^2 - a^2 + c^2)^2}{4c^2} =$$

$$= \frac{(2bc - b^2 + a^2 - c^2)(2bc + b^2 - a^2 + c^2)}{4c^2} =$$

$$= \frac{[a^2 - (b^2 - 2bc + c^2)][(b^2 + 2bc + c^2) - a^2]}{4c^2} =$$

$$= \frac{[a^2 - (b-c)^2][(b+c)^2 - a^2]}{4c^2} =$$

$$= \frac{(a - b + c)(a + b - c)(b + c - a)(b + c + a)}{4c^2} =$$

$$= \frac{(2P - 2b)(2P - 2c)(2P - 2a) \cdot 2P}{4c^2} =$$

$$= \frac{2(P-b)2(P-c)2(P-a) \cdot 2P}{4c^2} =$$

$$= \frac{4P(P-a)(P-b)(P-c)}{c^2}$$

$$\boxed{\begin{aligned} P &= \frac{a+b+c}{2} \\ 2P &= a+b+c \\ \rightarrow a+b &= 2P - c \\ a+c &= 2P - b \\ b+c &= 2P - a \end{aligned}}$$

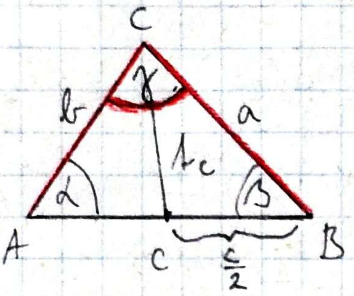
$$\Rightarrow \underline{S = \frac{c}{2} \cdot h_c} =$$

$$= \frac{c}{2} \cdot \sqrt{\frac{4P(P-a)(P-b)(P-c)}{c^2}} =$$

$$= \frac{c}{2} \cdot \frac{2\sqrt{P(P-a)(P-b)(P-c)}}{c} = \sqrt{P(P-a)(P-b)(P-c)}$$

$$\Rightarrow \boxed{S_\Delta = \sqrt{P(P-a)(P-b)(P-c)} \wedge P = \frac{a+b+c}{2}}$$

• $\triangle ABC: a=5;\ b=7;\ \gamma=60° \rightarrow c;\ \alpha;\ \beta;\ S;\ \rho;\ r;\ t_c = ?$



c) $c^2 = a^2 + b^2 - 2ab \cdot \cos(\gamma)$

$c^2 = 25 + 49 - 70 \cdot \cos(60°)$

$c = \sqrt{39}$

α) $\frac{\sin(\alpha)}{a} = \frac{\sin(\gamma)}{c}$

$\sin(\alpha) = \frac{5 \cdot \sin(60)}{\sqrt{39}} = \frac{\frac{5\sqrt{3}}{2}}{\sqrt{39}} = \frac{5\sqrt{3}}{2\sqrt{39}} = \frac{5}{2\sqrt{13}} = \frac{5\sqrt{13}}{26}$

$\alpha = 43{,}9°$

S) $S = \frac{1}{2} ab \sin(\gamma)$

$S = \frac{1}{2} \cdot 35 \cdot \frac{\sqrt{3}}{2} = \frac{35\sqrt{3}}{4} \doteq 15{,}15$

ρ) $S = \rho \cdot s$

$\rho = \frac{S}{\frac{a+b+c}{2}} = \frac{2 \cdot \frac{35\sqrt{3}}{4}}{5+7+\sqrt{39}} = \frac{35\sqrt{3}}{24+2\sqrt{39}} \doteq 1{,}66$

r) $2r = \frac{c}{\sin(\gamma)}$

$r = \frac{\sqrt{39}}{2 \cdot \frac{\sqrt{3}}{2}} = \sqrt{13}$

t_c) $t_c^2 = a^2 + \left(\frac{c}{2}\right)^2 - ac \cdot \cos(\beta)$

$t_c^2 = 25 + \frac{39}{4} - 5\sqrt{39} \cdot \cos(76{,}1)$

$t_c \doteq 5{,}22$

• $c \doteq \sqrt{39}$

• $\alpha \doteq 43{,}9°$

• $\beta \doteq 76{,}1$

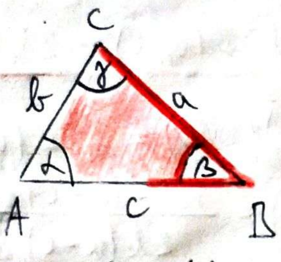
• $S \doteq 15{,}15$

• $\rho \doteq 1{,}66$

• $r = \sqrt{13}$

• $t_c \doteq 5{,}22$

• $\triangle ABC: S = 15;\ a = 3;\ \beta = 30^\circ \rightarrow b, c = ?$



c) $2S = a \cdot c \cdot \sin(\beta)$

$$c = \frac{2S}{a \cdot \sin(\beta)} = \frac{30}{3 \cdot \frac{1}{2}} = \underline{\underline{20}}$$

b) $b^2 = a^2 + c^2 - 2ac \cdot \cos(\beta)$

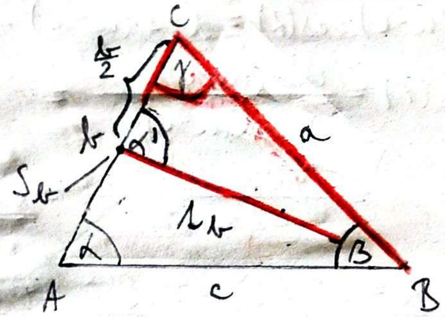
$$b^2 = 9 + 400 - 120 \cdot \frac{\sqrt{3}}{2} = 409 - 60\sqrt{3} \doteq 305$$

$\underline{\underline{b \doteq 17{,}47}}$

$b = 17{,}47$

$c = 20$

• $\triangle ABC: a = 5;\ t_b = 5;\ \gamma = 45^\circ \rightarrow b, c = ?$



b) $\triangle S_bBC$: $\dfrac{\frac{b}{2}}{\sin(180-(\alpha'+\gamma))} = \dfrac{t_b}{\sin(\gamma)}$

$$\frac{b}{2 \cdot \sin(\alpha'+\gamma)} = \frac{t_b}{\sin(\gamma)}$$

$$b = \frac{2 \cdot t_b \cdot \sin(\alpha'+\gamma)}{\sin(\gamma)}$$

$\rightarrow \alpha'$) $\triangle S_bBC: a = t_b \Rightarrow \underline{\underline{\alpha' = \gamma = 45^\circ}}$

$$b = \frac{10}{\frac{\sqrt{2}}{2}} = \frac{20}{\sqrt{2}} = \underline{\underline{10\sqrt{2}}}$$

$b = 10\sqrt{2}$

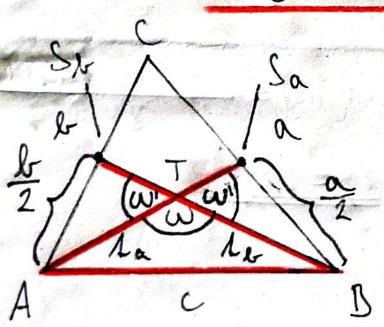
$c = 5\sqrt{5}$

c) $\triangle ABC$: $c^2 = a^2 + b^2 - 2ab\cos(\gamma)$

$$c^2 = 25 + 200 - 100\sqrt{2} \cdot \frac{\sqrt{2}}{2} = 225 - 100$$

$c = \sqrt{125} = \underline{\underline{5\sqrt{5}}}$

• $\triangle ABC: c = 8;\ t_a = 6;\ t_b = 9 \rightarrow a, b = ?$



ω') $\triangle ATB$: $c^2 = \left(\frac{2}{3}t_a\right)^2 + \left(\frac{2}{3}t_b\right)^2 - \frac{8}{9}t_a \cdot t_b \cdot \cos(\omega)$

$$\frac{8 \cdot t_a \cdot t_b \cdot \cos(\omega)}{9} = \frac{4t_a^2}{9} + \frac{4t_b^2}{9} - c^2$$

$$\cos(\omega) = \frac{4t_a^2 + 4t_b^2 - 9c^2}{8 \cdot t_a \cdot t_b} =$$

$$= \frac{4 \cdot 36 + 4 \cdot 81 - 9 \cdot 64}{8 \cdot 6 \cdot 9} = -\frac{1}{4}$$

$\underline{\underline{\omega \doteq 104{,}48^\circ}} \Rightarrow \underline{\underline{\omega' \doteq 75{,}52^\circ}}$

$a = 2\sqrt{34}$

$b = 2\sqrt{19}$

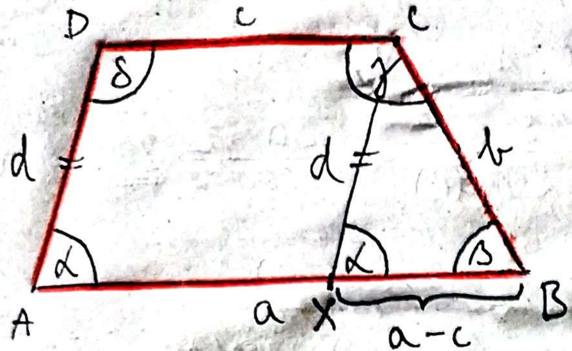
a) $\triangle TBS_a$: $\dfrac{a^2}{4} = \dfrac{t_a^2}{9} + \dfrac{4t_b^2}{9} - \dfrac{4t_a \cdot t_b \cdot \cos(\omega')}{9}$

$\underline{\underline{a = 2\sqrt{34}}} \Leftarrow a^2 = \dfrac{4(t_a^2 + 4t_b^2 - 4t_a \cdot t_b \cdot \frac{1}{4})}{9} = \dfrac{4(36 + 4 \cdot 81 - 54)}{9} = \dfrac{1224}{9}$

b) $\triangle TAS_b$: $\dfrac{b^2}{4} = \dfrac{t_b^2}{9} + \dfrac{4t_a^2}{9} - \dfrac{4t_a \cdot t_b \cdot \cos(\omega')}{9}$

$\underline{\underline{b = 2\sqrt{19}}} \Leftarrow b^2 = \dfrac{4(t_b^2 + 4t_a^2 - 4t_a t_b \cdot \cos(\omega'))}{9} = \dfrac{4(81 + 4 \cdot 36 - 54)}{9} = \dfrac{684}{9}$

• lichoběžník ABCD: $a = 30;\ b = 15;\ c = 20;\ d = 12 \to \alpha;\ \beta;\ \gamma;\ \delta$=



$\triangle XBC$: $\beta)\ d^2 = b^2 + (a-c)^2 - 2b(a-c)\cdot\cos(\beta)$

$$2b(a-c)\cos(\beta) = b^2 + (a-c)^2 - d^2$$

$$\cos(\beta) = \frac{b^2 + (a-c)^2 - d^2}{2b(a-c)} =$$

$$= \frac{225 + 100 - 144}{30\cdot 10} = \underline{\underline{0{,}60\overline{3}}}$$

$$\underline{\underline{\beta \doteq 52{,}89^\circ}} \Rightarrow \underline{\underline{\gamma \doteq 127{,}11^\circ}}$$

$\alpha)\ b^2 = d^2 + (a-c)^2 - 2d(a-c)\cdot\cos(\alpha)$

$$2d(a-c)\cos(\alpha) = d^2 + (a-c)^2 - b^2$$

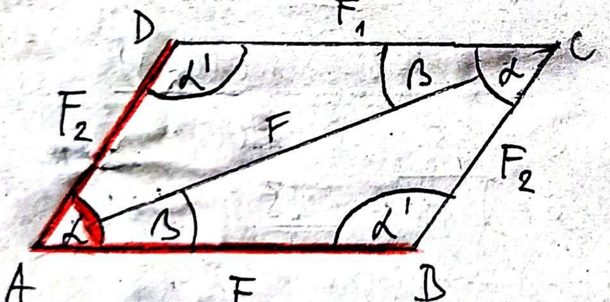
$$\cos(\alpha) = \frac{d^2 + (a-c)^2 - b^2}{2d(a-c)} =$$

$$= \frac{144 + 100 - 225}{24\cdot 10} = \underline{\underline{0{,}0791\overline{6}}}$$

$$\underline{\underline{\alpha \doteq 85{,}46^\circ}} \Rightarrow \underline{\underline{\delta \doteq 94{,}54^\circ}}$$

$\alpha = 85{,}46^\circ$
$\beta = 52{,}89^\circ$
$\gamma = 127{,}11^\circ$
$\delta = 94{,}54^\circ$

8) $F_1 = 58{,}6\,N;\ F_2 = 39{,}7\,N;\ \alpha = 65{,}3^\circ \to F;\ \beta = ?$



$F)\ \triangle ABC$: $F^2 = F_1^2 + F_2^2 - 2F_1F_2\cos(180-\alpha)$

$$F^2 = 58{,}6^2 + 39{,}7^2 - 2\cdot 58{,}6\cdot 39{,}7\cdot\cos(114{,}$$

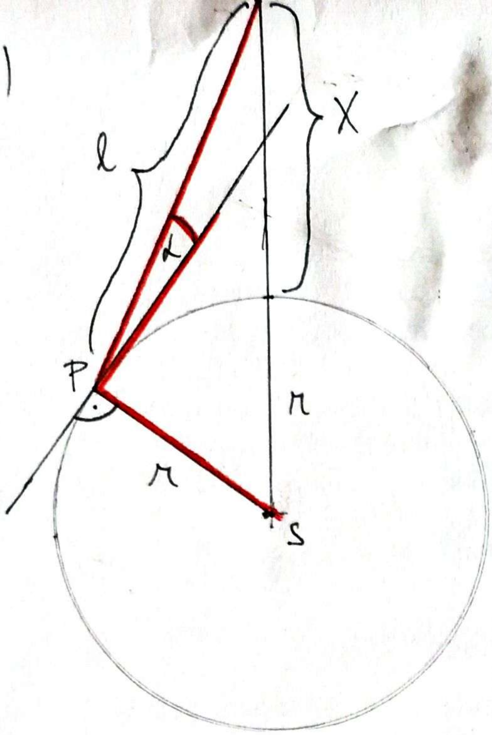
$$\underline{\underline{F \doteq 83{,}4\,N}}$$

$\beta)\ \triangle ABC$: $\dfrac{\sin(\beta)}{F_2} = \dfrac{\sin(180-\alpha)}{F}$

$$\sin(\beta) = \frac{39{,}7\cdot\sin(114{,}7)}{83{,}4} \doteq 0{,}4$$

$$\underline{\underline{\beta \doteq 25{,}6^\circ}}$$

1,



$l = 564$ km
$\alpha = 34{,}62^\circ$
$r = 6370$ km
$\Big\}\ x = ?$

$\triangle SPL: \ (x+r)^2 = r^2 + l^2 - 2rl\cdot\cos(\alpha+90)$

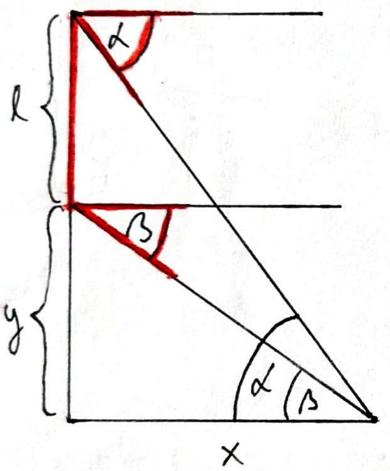
$$x = \sqrt{r^2 + l^2 - 2rl\cdot\cos(\alpha+90)} - r$$

$$x = \sqrt{6370^2 + 564^2 + 2\cdot 564\cdot 6370\cdot 0{,}57} - 6370$$

$$x \doteq 336{,}51 \text{ km}$$

2, sinová věta

3,



$l = 12$ m
$\alpha = 11{,}35^\circ$
$\beta = 5{,}75^\circ$
$\Big\}\ x = ?$

$$\mathrm{tg}(\alpha) = \frac{l+y}{x} \ \wedge \ \mathrm{tg}(\beta) = \frac{y}{x}$$

$$x = \frac{l+y}{\mathrm{tg}(\alpha)} \qquad x = \frac{y}{\mathrm{tg}(\beta)} \quad \rightarrow x = \frac{\frac{-l\cdot\mathrm{tg}(\beta)}{\mathrm{tg}(\beta)-\mathrm{tg}(\alpha)}}{\mathrm{tg}(\beta)}$$

$$\rightarrow \frac{l+y}{\mathrm{tg}(\alpha)} = \frac{y}{\mathrm{tg}(\beta)}$$

$$l\cdot\mathrm{tg}(\beta) + y\cdot\mathrm{tg}(\beta) = y\cdot\mathrm{tg}(\alpha)$$

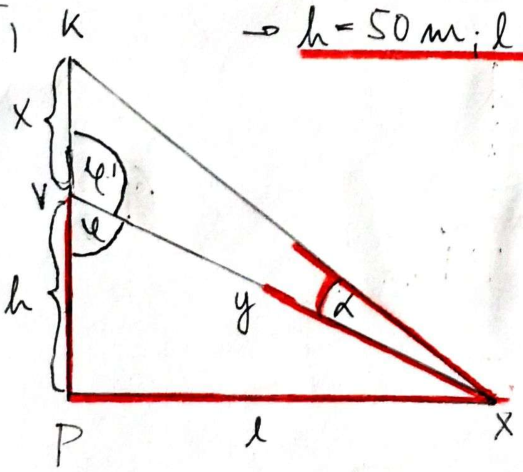
$$y\,(\mathrm{tg}(\beta) - \mathrm{tg}(\alpha)) = -l\cdot\mathrm{tg}(\beta)$$

$$y = \frac{-l\cdot\mathrm{tg}(\beta)}{\mathrm{tg}(\beta)-\mathrm{tg}(\alpha)}$$

$$x = \frac{-l}{\mathrm{tg}(\beta)-\mathrm{tg}(\alpha)}$$

$$x \doteq 119{,}96 \text{ m}$$

5,



$\rightarrow h = 50$ m; $l = 80$ m; $\alpha = 1{,}27^\circ \rightarrow x = ?$

• $\triangle PXV$: • $\mathrm{tg}(\varphi) = \frac{l}{h} = \frac{8}{5}$

$\varphi \doteq 58^\circ \Rightarrow \varphi' \doteq 122^\circ$

• $y = \sqrt{h^2 + l^2}$

$y = \sqrt{2500 + 6400}$

$y = 10\sqrt{89}$

• $\triangle VXK$:

$$\frac{x}{\sin(\alpha)} = \frac{y}{\sin(180-(\alpha+\varphi'))}$$

$$x = \frac{y\cdot\sin(\alpha)}{\sin(\alpha+\varphi')}$$

$$x = \frac{10\sqrt{89}\cdot\sin(1{,}27)}{\sin(123{,}27)}$$

$$x = 2{,}5 \text{ m}$$