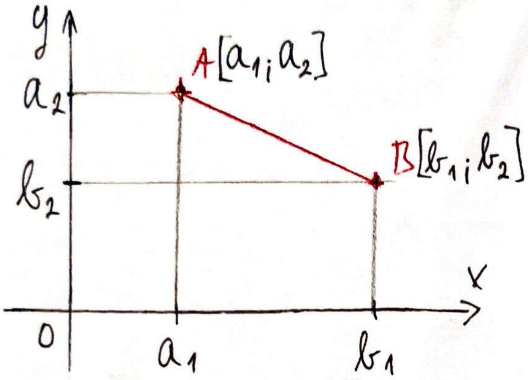
ANALITICKÁ GEOMETRIE

→ soustava souřadná v Π_1 - přímka

→ soustava souřadná v Π_2 - plocha

→ kartézská soustava souřadná - osy jsou na sebe kolmé
- jednotky na osách jsou stejné



$$\Rightarrow |AB|^2 = (a_1 - b_1)^2 + (a_2 - b_2)^2$$

$$|AB| = \sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2}$$

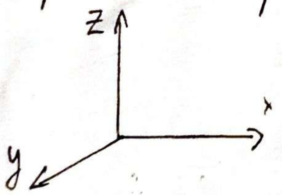
→ soustava souřadná v Π_3 - prostor

→ osy jsou na sebe kolmé a jednotky na osách jsou stejné

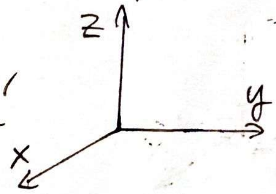
→ dlaň ruky nastavíme k ose $\vec{x}$ tak, aby prsty ukazovaly k ose $\vec{y}$ a vztyčený palec měl shodný směr s osou $\vec{z}$

→ pokud to splňuje levá ruka ⇒ levotočivá soustava

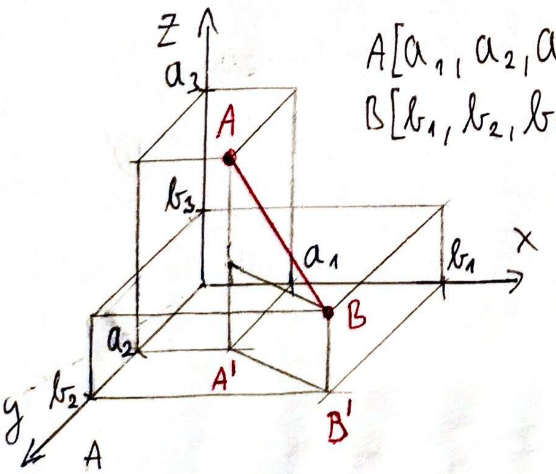
→ pokud to splňuje pravá ruka ⇒ pravotočivá soustava



levotočivá



pravotočivá

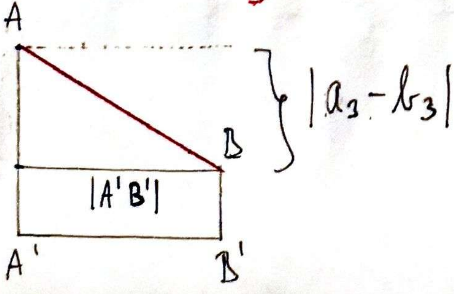


A[a_1, a_2, a_3]
B[b_1, b_2, b_3]

$$|AB|^2 = |A'B'|^2 + (a_3 - b_3)^2$$

$$|AB|^2 = (a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_3 - b_3)^2$$

$$\Rightarrow |AB| = \sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_3 - b_3)^2}$$



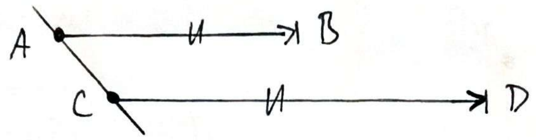
→ vektory

- → matematický vektor odpovídá posunutí do určitého směru
- → orientovaná úsečka $\overrightarrow{AB}$ má počáteční bod A a koncový bod B

$A \longmapsto B$ velikost $\overrightarrow{AB}$ je $|AB|$

→ souhlasně orientované úsečky

- → pokud jsou rovnoběžné a patří témuž směru

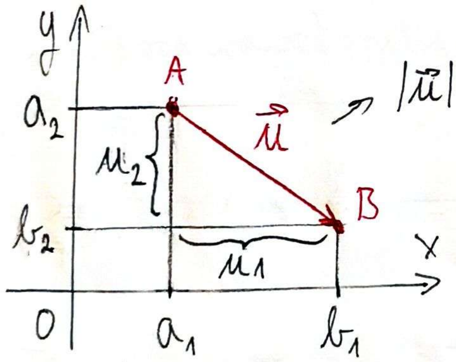


⇒ vektor je množina všech souhlasně orientovaných úseček stejné velikosti (délky)

⇒ vyberu si jednu z těch úseček → bude reprezentovat vektor

- → má počáteční bod A
  - ⇒ nazýváme jí umístění vektoru do bodu A

→ souřadnice vektoru v rovině – vektor pojmenujeme $\vec{u}$



↗ $|\vec{u}| = |AB|$

$$\left.\begin{aligned} u_1 &= b_1 - a_1 \\ u_2 &= b_2 - a_2 \end{aligned}\right\}$$

⇒ pokud bychom vektor $\vec{u}$ umístili do počátku, pak by koncový bod byl $[u_1; u_2]$

souřadnice vektoru zapisujeme

$$\vec{u}(u_1; u_2) = \vec{u}(b_1 - a_1; b_2 - a_2)$$

umístění vektoru $\overrightarrow{AB}$ zapisujeme

$\vec{u} = B - A$ → kompaktní zápis této soustavy rovnic

↳ říká to, že $u_n = b_n - a_n$

→ opačný vektor k vektoru $\vec{u}(u_1; u_2)$

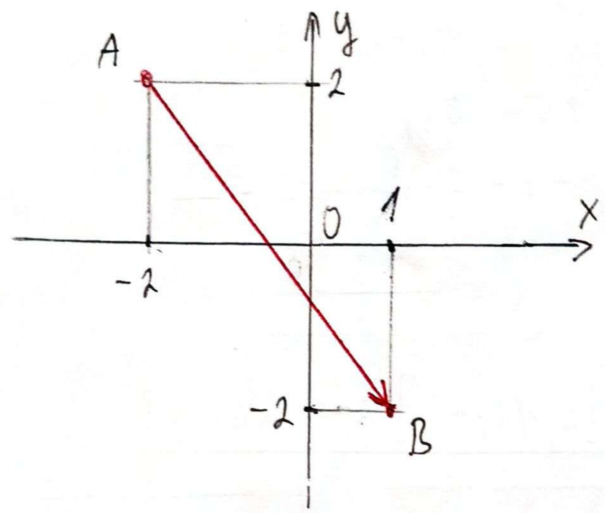
⇒ $-\vec{u}(-u_1; -u_2)$ – opačně orientovaný + stejně dlouhý

→ příklad

$\vec{u} = B - A \;\wedge\; B = [1; -2] \;\wedge\; \vec{u}(3; -4) \rightarrow A = ?$

$\Rightarrow \vec{u} = B - A \Rightarrow A = B - \vec{u}$

$$\left.\begin{array}{l} \bullet\ a_1 = b_1 - u_1 = 1 - 3 = -2 \\ \bullet\ a_2 = b_2 - u_2 = -2 + 4 = 2 \end{array}\right\} A = [-2; 2]$$

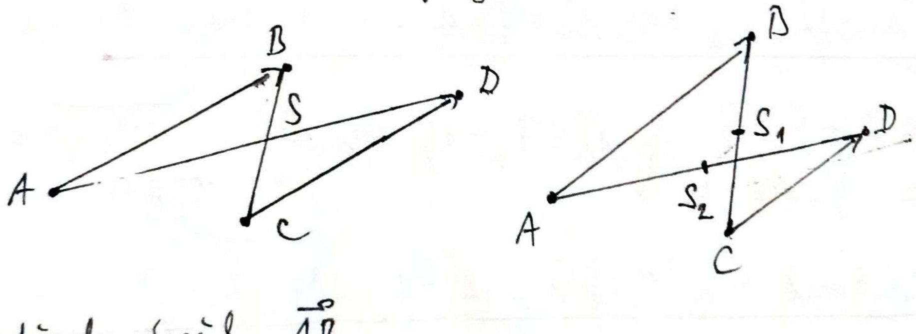


$$|\vec{u}| = |AB| = \sqrt{u_1^2 + u_2^2 + u_3^2}$$

→ nulový vektor

→ když $A \equiv B \Rightarrow \vec{o}(0;0)$

→ $\vec{AB}$ a $\vec{CD}$ určují stejný vektor, pokud $S(AD) = S(BC)$



→ střed úsečky $\vec{AB}$

→ $A + \vec{u} = B$ → pomocí $\vec{u}$ se z A dostanu do B

→ $A + \frac{1}{2}\vec{u} = S$ → pomocí poloviny $\vec{u}$ se z A dostanu na půl cesty do B

$$\Rightarrow S = A + \frac{1}{2}(B - A)$$

$$S = \frac{1}{2}(A + B)$$

→ příklady

- Na ose y urči bod, který má 3x větší vzdálenost od A než B

$A[5;-1] \quad B[0;-1] \Rightarrow Y[0;p] \wedge |AY| = 3 \cdot |BY|$

$$\sqrt{(a_1-y_1)^2+(a_2-y_2)^2} = 3\sqrt{(b_1-y_1)^2+(b_2-y_2)^2}$$

$$5^2+(1+p)^2 = 9(1+p)^2$$

$$25 = 8(p+1)^2$$

$$p+1 = \pm\frac{5}{2\sqrt{2}} = \pm\frac{5\sqrt{2}}{4}$$

$$p = \frac{-4\pm5\sqrt{2}}{4}$$

- 1, b, $K[5;1;7] \wedge L[2;11;-4] \Rightarrow |KL| = ?$

$$|KL| = \sqrt{3^2+10^2+11^2} = \sqrt{9+121+100} = \sqrt{230} \doteq 15{,}16$$

2, a, $A[3;p] \wedge B[-1;0] \wedge |AB| = 3\sqrt{2} \Rightarrow p = ?$

$$\sqrt{4^2+p^2} = 3\sqrt{2}$$

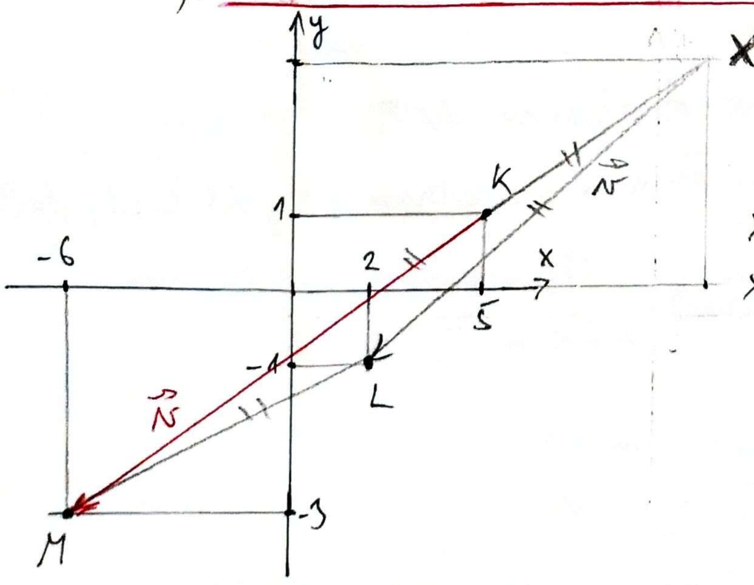
$$16+p^2 = 18 \Rightarrow p^2 = 2 \Rightarrow p = \pm\sqrt{2}$$

5, a, $A[-5;1] \wedge B[2;-3] \Rightarrow \vec{u} = B-A = ? \wedge |\vec{u}| = ?$

$$\left.\begin{aligned} u_1 &= 2+5 = 7 \\ u_2 &= -3-1 = -4 \end{aligned}\right\} \vec{u}(7;-4) \Rightarrow |\vec{u}| = \sqrt{7^2+4^2} = \sqrt{49+16}$$

$$|\vec{u}| = \sqrt{65}$$

9, KMLX - rovnoběžník $\wedge K[5;1] \wedge L[2;-1] \wedge M[-6;-3] \Rightarrow X = ?$



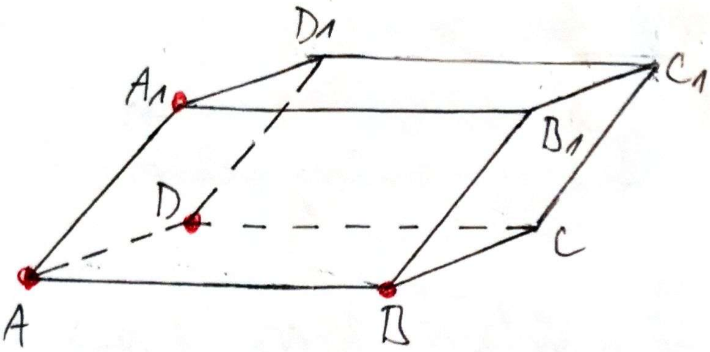
$$\vec{v} = \vec{v}$$

$$M-K = L-X \Rightarrow X = L+K-M$$

$$\left.\begin{aligned} x_1 &= 2+5+6 = 11 \\ x_2 &= -1+1+3 = 3 \end{aligned}\right\} X[11;3]$$

4) rovnobežnostěn $ABCDA_1B_1C_1D_1$

$A[2;-3;1] \wedge B[3;-4;2]; D[4;2;-3]; A_1[5;3;4] \Rightarrow C, B_1, C_1, D_1 = ?$



$\vec{u} = D - A$

$\vec{v} = A_1 - A$

$\vec{w} = B - A$

• D_1: $\left.\begin{array}{l} A + \vec{u} = D \\ A_1 + \vec{u} = D_1 \end{array}\right\} D_1 - D = A_1 - A \Rightarrow D_1 = D + A_1 - A$

$\Rightarrow D_1[4+5-2; 2+3+3; -3+4-1] \Rightarrow \underline{\underline{D_1[7;8;0]}}$

• B_1: $\left.\begin{array}{l} A + \vec{w} = B \\ A_1 + \vec{w} = B_1 \end{array}\right\} B_1 - B = A_1 - A \Rightarrow B_1 = B + A_1 - A$

$\Rightarrow B_1[3+5-2; -4+3+3; 2+4-1] \Rightarrow \underline{\underline{B_1[6;2;5]}}$

• C_1: $\left.\begin{array}{l} A + \vec{w} = B \\ D_1 + \vec{w} = C_1 \end{array}\right\} C_1 - B = D_1 - A \Rightarrow C_1 = B + D_1 - A$

$\Rightarrow C_1[3+7-2; -4+8+3; 2-1] \Rightarrow \underline{\underline{C_1[8;7;1]}}$

• C: $\left.\begin{array}{l} A + \vec{w} = B \\ D + \vec{w} = C \end{array}\right\} C - B = D - A \Rightarrow C = B + D - A$

$\Rightarrow C[3+4-2; -4+2+3; 2-3-1] \Rightarrow \underline{\underline{C[5;1;-2]}}$

→ Operace s vektory

→ násobení vektoru reálným číslem

→ pokud $\vec{v} = \vec{AV} \wedge k \in R$; potom $\vec{w} = k \cdot \vec{v}$ má umístění $\vec{w} = \vec{AW}$ a platí $|AW| = |k| \cdot |AV|$ a je-li

$k > 0 \Rightarrow W \in \mapsto AV$
$k < 0 \Rightarrow W \in \leftarrow\!\!\!\dashv AV$
} $k = -1$ - opačný vektor
$k = 0$ - nulový vektor

$\vec{v}(v_1; v_2; v_3) \Rightarrow k \cdot \vec{v} = \vec{w}(k \cdot v_1; k \cdot v_2; k \cdot v_3)$

→ jestliže tři body A, B, C leží na přímce, pak jsou kolineární

⇒ body A, B, C jsou kolineární $\Leftrightarrow \vec{AC} = k \cdot \vec{AB} \wedge k \in \mathbb{R} - \{0\}$

⇒ vektory $\vec{v}, \vec{w}$ jsou kolineární $\Leftrightarrow \exists\, k \in R;\ \vec{v} = k \cdot \vec{w}$

→ příklady

• ukaž, že $\vec{a}(-4;5)$ a $\vec{b}[2;-2{,}5]$ leží na jedné přímce

$\vec{a} = k \cdot \vec{b} \rightarrow -4 = 2k \Rightarrow k = -2$
$5 = -2{,}5k \Rightarrow k = -2$
} k existuje ⇒ leží

• rozhodni, zda $A[3;3]$, $B[5;4]$, $C[7;5]$ leží na 1 přímce a najdi y_D aby $D[-3;y_D]$ ležel na přímce AB

$\vec{AC} = k \cdot \vec{AB}$

$\vec{AC}(4;2) = k \cdot \vec{AB}(2;1)$
$4 = 2k$
$2 = k$
} $k = 2 \Rightarrow$ leží

$\vec{AB} = l \cdot \vec{AD}$

$\vec{AB}(2;1) = l \cdot (-6; y_D - 3)$
$2 = -6l \Rightarrow l = -\frac{1}{3}$
$1 = l(y_D - 3)$
$-3 = y_D - 3 \Rightarrow \underline{y_D = 0}$

• $\vec{z}(6;y) \wedge |\vec{z}| = 10 \Rightarrow y = ?$

$10 = \sqrt{36 + y^2}$

$100 = 36 + y^2 \Rightarrow \underline{y = \pm 8}$

• $\vec{u}(7;-1) \wedge \vec{v} \parallel \vec{u} \wedge |\vec{v}| = 10 \Rightarrow v = ?$

$\vec{u} \cdot k = \vec{v} \quad \wedge \quad v_1^2 + v_2^2 = 100$

$7 \cdot k = v_1 \qquad 49k^2 + k^2 = 100$

$-1k = v_2 \qquad k^2 = 2$

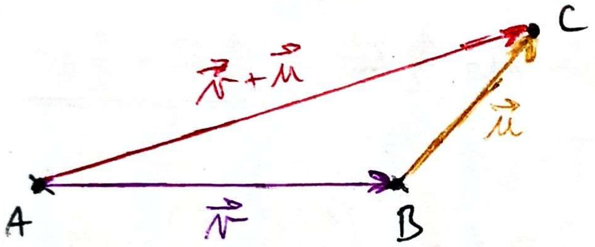
$k = \pm\sqrt{2}$

$v_1 = \pm 7\sqrt{2}$

$v_2 = \mp \sqrt{2} \quad \Rightarrow \quad \vec{v}_1(7\sqrt{2}; -\sqrt{2})$

$\vec{v}_2(-7\sqrt{2}; \sqrt{2})$

→ sčítání vektorů



→ nechť $\vec{v} = \vec{AB}$ a $\vec{u} = \vec{BC}$, pak $\vec{w} = \vec{AC} = \vec{v} + \vec{u}$

$\left.\begin{array}{l} \vec{v}(v_1; v_2; v_3) \\ \vec{u}(u_1; u_2; u_3) \end{array}\right\} \vec{w} = \vec{v} + \vec{u} = \vec{w}(v_1 + u_1; v_2 + u_2; v_3 + u_3)$

→ rozdíl: $\vec{v} - \vec{u} = \vec{v} + (-\vec{u})$ – vektor opačný

→ lineární kombinace vektorů

• jsou dány vektory $\vec{u}, \vec{v}, \vec{k}, \vec{w}$

⇒ pokud $\exists\, k, l, m \in R$; $k \cdot \vec{u} + l \cdot \vec{v} + m \cdot \vec{k} = \vec{w}$,

potom je $\vec{w}$ lineární kombinací vektorů $\vec{u}, \vec{v}, \vec{k}$

→ lineární kombinace v E_2

→ pokud $\vec{v} \nparallel \vec{u}$, pak můžu vytvořit jakékoli $\vec{w}$ 1 způsobem

→ pokud $\vec{v} \parallel \vec{w} \parallel \vec{u}$, pak můžu vytvořit $\vec{w}$ ∞ způsoby

→ pokud $\vec{v} \parallel \vec{u} \nparallel \vec{w}$, pak $\vec{w}$ nemůžu vytvořit

→ příklady

• $A[1;2]$, $B[3;5]$, $C[-2;4]$

a) dokaž, že existuje $\triangle ABC$ − A, B, C nesmí být kolineární

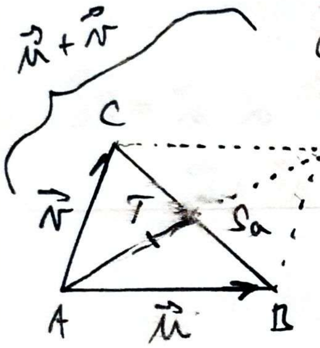
$$\vec{AB} = k \cdot \vec{AC}$$

$$\vec{AB}(2;3) = k \cdot \vec{AC}(-3;2) \quad / \quad \left.\begin{matrix} 2 = -3k \\ 3 = 2k \end{matrix}\right\} \text{ k neexistuje} \Rightarrow$$

$\Rightarrow \triangle ABC$ existuje

b) $\vec{u} = \vec{AB} \wedge \vec{v} = \vec{AC}$, zapiš pomocí $\vec{u}, \vec{v}$ vektor $\vec{w} = \vec{BC}$

$$\rightarrow -\vec{u} = \vec{BA} \Rightarrow \vec{BA} + \vec{AC} = \vec{BC} \Rightarrow \vec{w} = -\vec{u} + \vec{v}$$



c) urči $t_a = \vec{t} = \vec{AS_a}$ jako lin. kombinaci vektorů

$$\rightarrow \vec{t} = \frac{1}{2}(\vec{u}+\vec{v}) = \vec{u} + \frac{1}{2}\vec{w} = \frac{1}{2}\vec{u} + \frac{1}{2}\vec{v}$$

d) urči souřadnice T

$$\rightarrow \vec{AT} = \frac{2}{3} \cdot \vec{t} \Rightarrow T = A + \frac{2}{3}\left(\frac{1}{2}(\vec{u}+\vec{v})\right) = A + \frac{1}{3}(\vec{u}+\vec{v})$$

$$\Rightarrow T\left[\frac{2}{3}; \frac{11}{3}\right]$$

e) velikost stran a t_a

• $\vec{t_a} = \left(-\frac{1}{2}; \frac{5}{2}\right) \Rightarrow |t_a| = \sqrt{\frac{1}{4} + \frac{25}{4}} = \frac{\sqrt{26}}{2}$

• $c = |AB| = \sqrt{4+9} = \sqrt{13}$

• $b = |AC| = \sqrt{9+4} = \sqrt{13}$

• $a = |BC| \wedge \vec{BC}(-5;-1) \Rightarrow a = \sqrt{25+1} = \sqrt{26}$

d) $T = A + \frac{1}{3}(\vec{AB} + \vec{AC}) = A + \frac{1}{3}B - \frac{1}{3}A + \frac{1}{3}C - \frac{1}{3}A$

$$\Rightarrow T = \frac{1}{3}(A+B+C)$$

→ Těžiště

→ těžiště konvexního mnohoúhelníku je průměr jeho vrcholů

$$T_n = \frac{N_1 + N_2 + \ldots + N_n}{n}$$

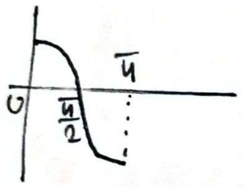
→ Skalární součin

→ odchylka vektorů – φ

– menší z úhlů, které svírají ty dva vektory, když je umístíme do stejného bodu

→ skalární součin vektorů $\vec{v}$, $\vec{u}$ je reálné číslo

$$\vec{u}\cdot\vec{v} = |\vec{u}|\cdot|\vec{v}|\cdot\cos(\varphi) \quad \wedge \quad \varphi \text{ je jejich odchylka}$$



- $\varphi \in \langle 0; 90)$ $\iff$ $\vec{u}\cdot\vec{v} > 0$
- $\varphi \in (90; 180)$ $\iff$ $\vec{u}\cdot\vec{v} < 0$
- $\varphi = 90^\circ$ $\Rightarrow$ $\vec{u}\cdot\vec{v} = 0$

$\wedge \ \vec{u}\cdot\vec{v} \neq \vec{o}(0;0)$

- $\vec{u} = \vec{o} \vee \vec{v} = \vec{o} \Rightarrow \vec{u}\cdot\vec{v} = 0$

$$\Rightarrow \vec{u}\cdot\vec{v} = 0 \iff (\vec{u} \perp \vec{v}) \vee (\vec{u} = \vec{o} \vee \vec{v} = \vec{o})$$

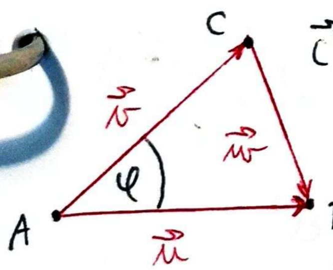
→ kolmice na vektor → π_2

→ vektor $\vec{u}$ je kolmý na vektor $\vec{v}(a;b)$ $\iff$

$$\vec{u} = k\cdot(-b;a) \quad \wedge \quad k \in R \setminus \{0\}$$

⇒ prohodím souřadnice a u jedné z nich změním znaménko

→ odchylka vektorů pomocí skalárního součinu



$\vec{CB} = \vec{w} = -\vec{v} + \vec{u} \ \wedge \ \vec{v} = \vec{AC} \ \wedge \ \vec{u} = \vec{AB}$

→ Kos. v.: $|\vec{w}|^2 = |\vec{u}|^2 + |\vec{v}|^2 - \overbrace{2|\vec{u}|\cdot|\vec{v}|\cdot\cos(\varphi)}^{2\cdot(\vec{u}\cdot\vec{v})}$

$$\Rightarrow \vec{u}\cdot\vec{v} = \frac{1}{2}\left(|\vec{u}|^2 + |\vec{v}|^2 - |\vec{w}|^2\right)$$

$$\Rightarrow \vec{u}\cdot\vec{v} = \frac{1}{2}\left(u_1^2 + u_2^2 + v_1^2 + v_2^2 - |\vec{u}-\vec{v}|^2\right) =$$

$$= \frac{1}{2}\left(u_1^2 + u_2^2 + v_1^2 + v_2^2 - (u_1-v_1)^2 - (u_2-v_2)^2\right) =$$

$$= \frac{1}{2}\left(u_1^2 + u_2^2 + v_1^2 + v_2^2 - u_1^2 + 2u_1v_1 - v_1^2 - u_2^2 + 2u_2v_2 - v_2^2\right) =$$

$$= \frac{1}{2}\left(2u_1v_1 + 2u_2v_2\right) \Rightarrow \vec{u}\cdot\vec{v} = u_1\cdot v_1 + u_2\cdot v_2 + u_3\cdot v_3 + \dots$$

$$\Rightarrow \cos(\varphi) = \frac{\vec{u}\cdot\vec{v}}{|\vec{u}|\cdot|\vec{v}|} = \frac{u_1\cdot v_1 + u_2\cdot v_2}{|\vec{u}|\cdot|\vec{v}|}$$

→ příklady

• $\vec{u}(\lambda; 2; -1), \vec{v}(1; -1; 3) \wedge \vec{u} \perp \vec{v} \Rightarrow \lambda = ?$

$\vec{u} \perp \vec{v} \Rightarrow \vec{u} \cdot \vec{v} = 0 = \lambda - 2\lambda - 3 = -\lambda - 3 \Rightarrow \underline{\underline{\lambda = -3}}$

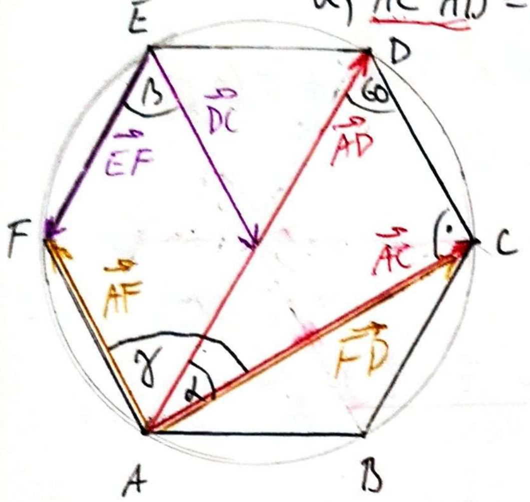
• $\vec{u}(1; 1; -1), \vec{v}(2; 1; 3) \Rightarrow \varphi = ?$

$\vec{u} \cdot \vec{v} = 2 + 1 - 3 = 0 \Rightarrow \varphi = 90°$

• pravidelný šestiúhelník, strana $a = 2$

a) $\vec{AC} \cdot \vec{AD} = ?$ b) $\vec{DC} \cdot \vec{EF} = ?$ c) $\vec{FD} \cdot \vec{AF} = ?$

Tk
↓



a) $\vec{AC} \cdot \vec{AD} = |\vec{AC}| \cdot |\vec{AD}| \cdot \cos(\alpha) \rightarrow |\vec{AD}| = 2a \wedge \alpha = 30°$

$\Rightarrow |\vec{AC}| = \sqrt{|\vec{AD}|^2 - a^2} = \sqrt{4a^2 - a^2} = a\sqrt{3}$

$\Rightarrow \vec{AC} \cdot \vec{AD} = 2a^2\sqrt{3} \cdot \frac{\sqrt{3}}{2} = 3a^2 = \underline{\underline{12}}$

b) $\vec{DC} \cdot \vec{EF} = |\vec{DC}| \cdot |\vec{EF}| \cdot \cos(\beta) = a \cdot a \cdot \frac{1}{2} = \underline{\underline{2}}$

c) $\gamma = 90° \Rightarrow \vec{FD} \cdot \vec{AF} = \underline{\underline{0}}$

• $|\vec{v}| = 4 \wedge \vec{u}(\sqrt{3}; -1) \wedge \varphi = 30° \Rightarrow \vec{v} = ?$

$4 \cdot \sqrt{3+1} \cdot \cos(30) = v_1\sqrt{3} - v_2$

$8 \cdot \frac{\sqrt{3}}{2} = v_1\sqrt{3} - v_2$

$v_2 = v_1\sqrt{3} - 4\sqrt{3} = \sqrt{v_1^2 \cdot 3 - 8 \cdot 3 \cdot v_1 + 16 \cdot 3}$

$4 = \sqrt{v_1^2 + v_2^2} = \sqrt{v_1^2 + 3v_1^2 - 24v_1 + 48}$

$16 = 4v_1^2 - 24v_1 + 48$

$\underline{v_1^2 - 6v_1 + 8 = 0} \rightarrow v_1 = 2 \Rightarrow v_2 = -2\sqrt{3}$

$\rightarrow v_1 = 4 \Rightarrow v_2 = 0$

$\Rightarrow \underline{\underline{\vec{v_1}(2; -2\sqrt{3}) \ \vec{v_2}(4; 0)}}$

• $A[2; 5; 10] \wedge B[2; 1; 7] \wedge X[a; 0; 0] \wedge |\sphericalangle ABX| = 60° \Rightarrow a = ?$

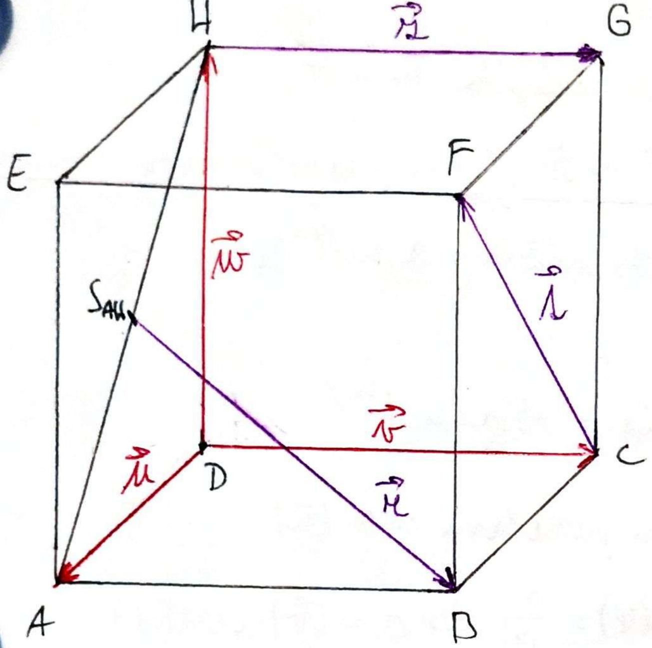
$\vec{BA} = (0; 4; 3) \Rightarrow |\vec{BA}| = \sqrt{16+9} = 5 \ | \ \vec{BX} = (a-2; -1; -7) \Rightarrow |\vec{BX}| = \sqrt{a^2 - 4a + 4 + 1 + 49}$

$\rightarrow \vec{BA} \cdot \vec{BX} = 5 \cdot \sqrt{a^2 - 4a + 54} \cdot \frac{1}{2} = 0 - 4 - 21 = -25$

$\sqrt{a^2 - 4a + 54} = -10$

$\Rightarrow$ $\underline{\underline{a \text{ neexistuje}}}$

• $\vec{u} = \overrightarrow{DA} \wedge \vec{v} = \overrightarrow{DC} \wedge \vec{w} = \overrightarrow{DH}$ → zapiš vektory:
$\vec{s} = \overrightarrow{CF} \wedge \vec{z} = \overrightarrow{HG} \wedge \vec{r} = \overrightarrow{S_{AH}B}$ jako lin. komb. $\vec{u}, \vec{v}, \vec{w}$.



$\vec{s} = \vec{u} + \vec{w}$

$\vec{z} = \vec{v}$

$\vec{r} = \frac{1}{2}\vec{u} - \frac{1}{2}\vec{w} + \vec{v}$

$\vec{r} = \frac{1}{2}\vec{s} - \vec{w} + \vec{v} = \frac{1}{2}\vec{u} - \frac{1}{2}\vec{w} + \vec{v}$

→ vlastnosti skalárního součinu

• $\vec{u} \cdot \vec{v} = \vec{v} \cdot \vec{u}$

• $(k \cdot \vec{u}) \cdot \vec{v} = k \cdot (\vec{u} \cdot \vec{v})$

• $\vec{u}(\vec{v} + \vec{w}) = \vec{u} \cdot \vec{v} + \vec{u} \cdot \vec{w}$

LZ = lin. závislé
NZ = lin. nezávislé

→ lineární závislost vektorů

→ skupina vektorů je lineárně závislá, pokud je jeden z nich lineární kombinací těch ostatních

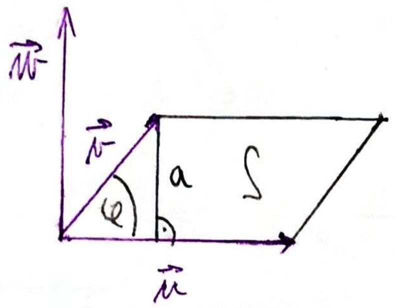
⇒ $\vec{u}, \vec{v}, \vec{w}$ jsou LZ $\Leftrightarrow \vec{w} = k \cdot \vec{u} + l \cdot \vec{v}$

→ vektorový součin - pouze v Π_3

⇒ vektorový součin vektorů $\vec{u}$ a $\vec{v}$ je $\vec{w} = \vec{u} \times \vec{v}$

- $\vec{w} \perp \vec{u} \wedge \vec{w} \perp \vec{v}$
- $|\vec{w}| = |\vec{u}| \cdot |\vec{v}| \cdot \sin(\varphi)$ - φ = odchylka $\vec{u}$ a $\vec{v}$
- vektory $\vec{u} \sim \vec{x}$, $\vec{v} \sim \vec{y}$, $\vec{w} \sim \vec{z}$ tvoří pravotočivou soustavu
- jsou-li $\vec{u}, \vec{v}$ kolineární nebo některý z nich je nulový, potom $\vec{u} \times \vec{v} = \vec{o}(0;0)$

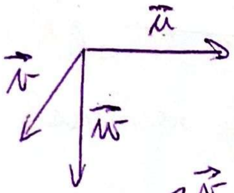
⇒ vektory $\vec{u}, \vec{v}$ tvoří rovnoběžník o obsahu $|\vec{w}|$



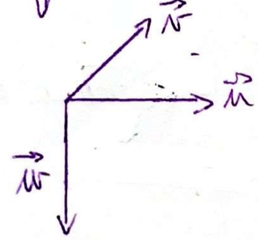
S = výška · podstava $= a \cdot |\vec{u}|$

$\sin(\varphi) = \frac{a}{|\vec{v}|} \Rightarrow a = |\vec{v}| \cdot \sin(\varphi)$

$\Rightarrow S = |\vec{u}| \cdot |\vec{v}| \cdot \sin(\varphi)$



$\vec{w} = \vec{u} \times (-\vec{v}) \Rightarrow \vec{u} \times (-\vec{v}) = -(\vec{u} \times \vec{v})$



$\vec{w} = \vec{v} \times \vec{u} \Rightarrow \vec{v} \times \vec{u} = -(\vec{u} \times \vec{v})$

$|k \cdot \vec{u}| = \sqrt{k^2 u_1^2 + k^2 u_2^2} = |k| \cdot \sqrt{u_1^2 + u_2^2} = |k| \cdot |\vec{u}|$

$\Rightarrow \vec{w}_1 = \vec{u} \times \vec{v} \rightarrow |\vec{w}_1| = |\vec{u}| \cdot |\vec{v}| \cdot \sin(\varphi)$

$\vec{w}_2 = (k \cdot \vec{u}) \times \vec{v} \rightarrow |\vec{w}_2| = |k| \cdot |\vec{u}| \cdot |\vec{v}| \cdot \sin(\varphi)$

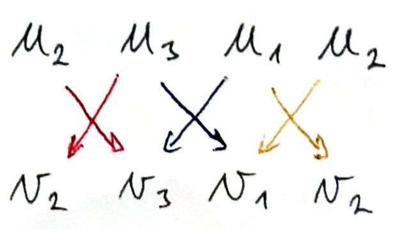
$\Rightarrow |(k \cdot \vec{u}) \times \vec{v}| = |k| \cdot |\vec{u} \times \vec{v}| \rightarrow |\vec{w}_2| = |k| \cdot |\vec{w}_1|$

→ $k > 0 \rightarrow w_1, w_2$ - opačná orientace

→ $k < 0 \rightarrow w_1, w_2$ - opačná orientace

→ souřadnice $\vec{w} = \vec{u} \times \vec{v}$

- $w_1 = u_2 \cdot v_3 - u_3 \cdot v_2$
- $w_2 = u_3 \cdot v_1 - u_1 \cdot v_3$
- $w_3 = u_1 \cdot v_2 - u_2 \cdot v_1$



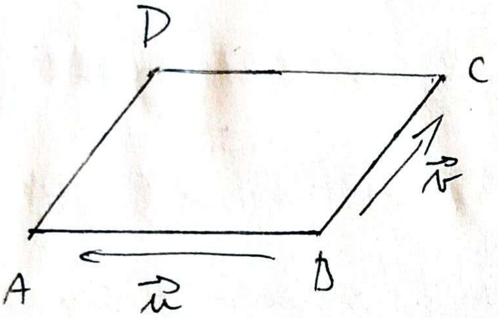
→ vektorový súčin v π_2

→ ku každému bodu pridám tretiu súradnicu $z = 0$

$$\left.\begin{array}{l}\vec{u}(u_1, u_2, 0)\\ \vec{v}(v_1, v_2, 0)\end{array}\right\} \vec{u} \times \vec{v} = \vec{w}(0; 0; z) \Rightarrow |\vec{w}| = |z|$$

→ príklady

- $A[0;0;0]$ $B[3;0;1]$ $C[5;2;9]$ → $S(ABCD) = ?$



$S(ABCD) = |\vec{u} \times \vec{v}| = |\vec{w}|$

$\vec{u} = (-3; 0; -1)$ $-3; 0$

$\vec{v} = (2; 2; 8)$ $2; 2$

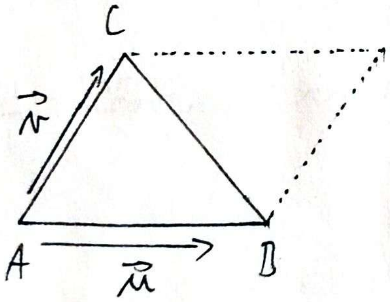
$w_1 = 0 + 2 = 2$

$w_2 = -2 + 24 = 22$

$w_3 = -6 - 0 = -6$

$\Rightarrow \vec{w} = (2; 22; -6) \Rightarrow S(ABCD) = \sqrt{4 + 22^2 + 36} = \sqrt{524} \doteq 23$

- $A[-7;1;2]$ $B[1;-4;0]$ $C[2;5;3]$ → $S(ABC) = ?$



$S(ABC) = \frac{1}{2}|\vec{u} \times \vec{v}| = \frac{1}{2}|\vec{w}|$

$\vec{u} = (8; -5; -2)$ $8; -5$

$\vec{v} = (9; 4; 1)$ $9; 4$

$w_1 = -5 + 8 = 3$

$w_2 = -18 - 8 = -26$

$w_3 = 32 + 45 = 77$

$\Rightarrow \vec{w} = (3; -26; 77) \Rightarrow S(ABC) = \frac{1}{2}\sqrt{9 + 26^2 + 77^2} \doteq 40{,}7$

- $A[2;-1]$ $B[-1;4]$ $C[3;-2]$ → $S(ABC) = ?$

$A[2;-1] = [2;-1;0]$

$B[-1;4] = [-1;4;0]$

$C[3;-2] = [3;-2;0]$

$\vec{AB} = \vec{u}(-3; 5; 0)$

$\vec{AC} = \vec{v}(1; -1; 0)$

$w_1 = 0 \wedge w_2 = 0$

$w_3 = 3 - 5 = -2$

$\Rightarrow \vec{u} \times \vec{v} = (0; 0; -2)$

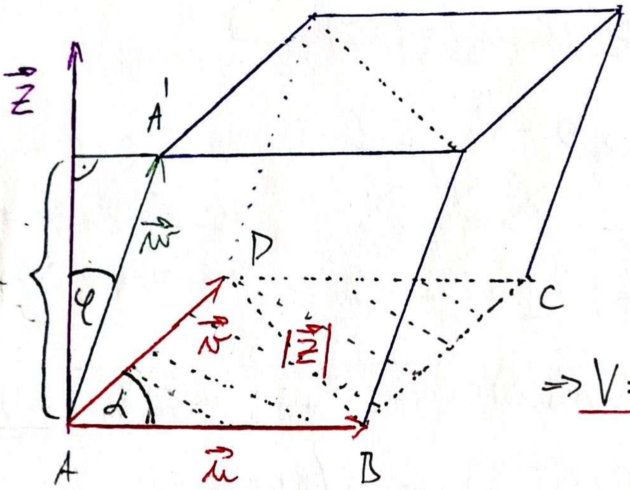
$S(ABC) = \frac{1}{2}\sqrt{4} = 1$

⇒ Smíšený součin

→ Nechť jsou dány vektory $\vec{u}, \vec{v}, \vec{w} \subset \Pi_3$, potom je smíšený součin těchto vektorů je skalární součin jednoho z těchto vektorů a vektorovým součinem zbývajících dvou. Smíšený součin je číslo

$$\vec{w}\cdot(\vec{u}\times\vec{v}) = (\vec{u}\times\vec{v})\cdot\vec{w}$$

→ $(\vec{u}\times\vec{v}) = \vec{Z} \Rightarrow \vec{Z}\cdot\vec{w} = |\vec{Z}|\cdot|\vec{w}|\cdot\cos(\varphi)$



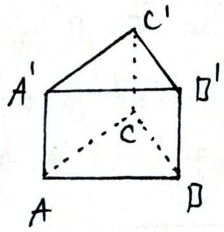
→ $\cos\varphi = \frac{a}{|\vec{w}|} \Rightarrow a = |\vec{w}|\cdot\cos(\varphi)$

$\Rightarrow (\vec{u}\times\vec{v})\cdot\vec{w} = |\vec{Z}|\cdot|\vec{w}|\cdot\cos\varphi = |\vec{Z}|\cdot a$

⇒ podstava · výška = objem

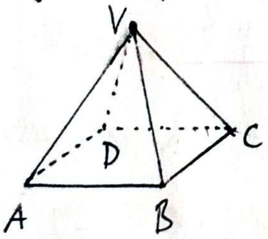
$$\Rightarrow V = |\vec{w}\cdot(\vec{u}\times\vec{v})| = |\vec{v}\cdot(\vec{u}\times\vec{w})| = |\vec{u}\cdot(\vec{v}\times\vec{w})|$$

- objem rovnoběžnostěnu: $V = |(\vec{AB}\times\vec{AD})\cdot\vec{AA'}|$
- objem kosého trojbokého hranolu → polovina rovnoběžnostěnu



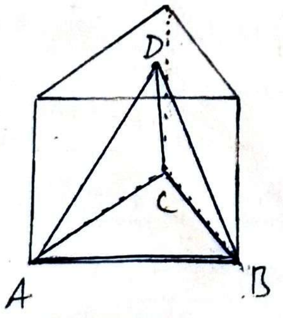
$$\Rightarrow V = \frac{1}{2}|(\vec{AB}\times\vec{AC})\cdot\vec{AA'}|$$

- objem čtyřbokého jehlanu



$$V = \frac{1}{3}|(\vec{AB}\times\vec{AD})\cdot\vec{AV}|$$

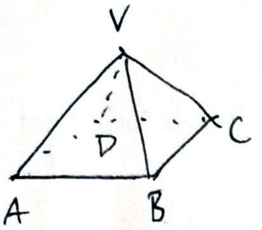
- objem čtyřstěnu → jehlan z trojbokého hranolu ⇒ $\frac{1}{3}$ objemu



$$\Rightarrow V = \frac{1}{6}|(\vec{AB}\times\vec{AC})\cdot\vec{AD}|$$

→ *příklady*

• $A[2;3;4]$ $B[-1;4;-2]$ $D[0;2;-5]$ $V[3;2;1]$ $\Rightarrow V(ABCDV)=?$



$$\left.\begin{array}{l}\vec{AB}=(-3;1;-6)\ -3,1\\ \vec{AD}=(-2;-1;-9)\ -2,-1\end{array}\right\}\begin{array}{l}z_1=-9-6\\ z_2=12-27\\ z_3=3+2\end{array}\Bigg\}\ \vec{z}(-15;-15;5)$$

$\vec{AV}=(1;-1;-3)$

$\Rightarrow V=\frac{1}{3}|(\vec{z}\cdot\vec{AV})|=\frac{1}{3}|-15+15-15|=\underline{\underline{5}}$

• $A[2;2;3]$ $B[6;3;0]$ $C[3;-1;-1]$ $X[a;0;0]$ $\wedge$ $V(ABCX)=26 \Rightarrow a=?$

$V=26=\frac{1}{6}|(\vec{AB}\times\vec{AC})\cdot\vec{AX}|$

$$\left.\begin{array}{l}\vec{AB}=(4;1;-3)\ 4;1\\ \vec{AC}=(1;-3;-4)\ 1;-3\end{array}\right\}\begin{array}{l}z_1=-4-9\\ z_2=-3+16\\ z_3=-12-1\end{array}\Bigg\}\ \vec{z}(-13;13;-13)$$

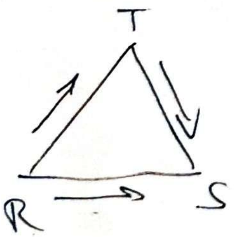
$\vec{AX}=(a-2;-2;-3)$

$\Rightarrow 26=\frac{1}{6}|-13a+26-26+39|=\frac{1}{6}|-13a+39|$

$\Rightarrow 156=|39-13a|=13|3-a| \Rightarrow 12=|3-a|$

$\Rightarrow \pm 12=3-a \Rightarrow a=3\pm 12 \Rightarrow \underline{\underline{a_1=15 \wedge a_2=-9}}$

• $R[4;1;0]$ $S[4;-2;-3]$ $T[1;-2;0]$ $\Rightarrow \Delta RST - o, S,$ úhly $=?$



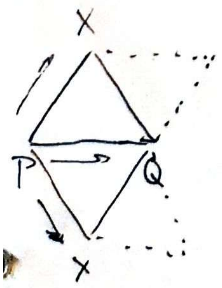
$\vec{RS}=(0;-3;-3) \Rightarrow |\vec{RS}|=|\vec{RT}|=|\vec{TS}|=\sqrt{9+9}=\sqrt{2\cdot 9}=3\sqrt{2}$

$\vec{RT}=(-3;-3;0)$ $\qquad \underline{\underline{o=9\sqrt{2}}}$ $\qquad \underline{\underline{\alpha=\beta=\gamma=60^\circ}}$

$\vec{TS}=(3;0;-3)$

$S=\frac{1}{2}3\sqrt{2}\cdot 3\sqrt{2}\cdot\sin(60)=\frac{1}{2}\cdot 18\cdot\frac{\sqrt{3}}{2}=\underline{\underline{\frac{9\sqrt{3}}{2}}}$

• $P[4;0]$ $Q[2;-4]$ $X[a;0]$ $\wedge$ $S(\Delta PQX)=3 \Rightarrow a=?$



$S(\Delta PQX)=3=\frac{1}{2}|\vec{PQ}\times\vec{PX}|=\frac{1}{2}|\vec{w}| \Rightarrow |\vec{w}|=6$

$$\left.\begin{array}{l}\vec{PQ}=(-2;-4;0)\\ \vec{PX}=(a-4;0;0)\end{array}\right\}\ \vec{w}=(0;0;4a-16) \Rightarrow |\vec{w}|=\sqrt{(4a-16)^2}=|4a-16|$$

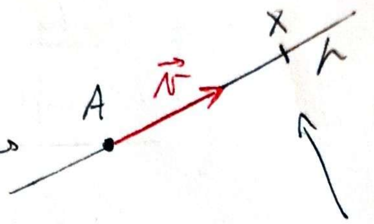
$\Rightarrow 6=|4a-16| \Rightarrow 4a-16=\pm 6$

$4a=16\pm 6$

$a=4\pm\frac{3}{2}$ $\qquad \underline{\underline{a_1=\frac{11}{2}}}$ $\qquad \underline{\underline{a_2=\frac{5}{2}}}$

→ Přímka – π_2

→ určení ⟨ 2 body → 

bodem a směrovým vektorem → 

→ Parametrické vyjádření přímky

→ na přímce p určené bodem A a směr. vekt. $\vec{v}$ běží bod X

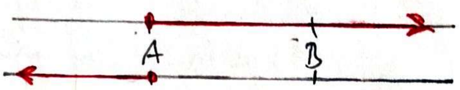
$$\Rightarrow p: X = A + t\cdot\vec{v}\,;\ t\in\mathbb{R}$$ → t = parametr bodu X

→ přímka p je množinou všech bodů X splňující tuto rovnici

→ vyjádření pomocí bodů A, B

$$\Rightarrow p: X = A + t\cdot\vec{AB} = A + t(B-A)\,;\ t\in\mathbb{R}$$

- $\overleftrightarrow{AB}: X = A + t\cdot\vec{AB}\,;\ t\in\mathbb{R}$
- $\vdash\!\dashv AB: X = A + t\cdot\vec{AB}\,;\ t\in\langle 0;1\rangle$ → $t=0: A \wedge t=1: B$
- $\mapsto AB: X = A + t\cdot\vec{AB}\,;\ t\in\langle 0;\infty)$
- $\leftarrow\!\dashv AB: X = A + t\cdot\vec{AB}\,;\ t\in(-\infty;0\rangle$



→ příklad

- $p: B[1;2\sqrt{3}],\ \vec{u}(5;-2) \rightarrow X[x;y] = ? \wedge P[2;\sqrt{3}]\in p = ?$

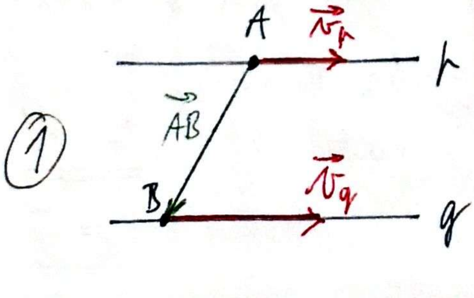
$X = B + t\cdot\vec{u}$

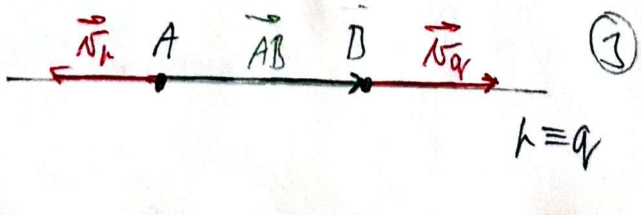
$$\left.\begin{aligned} x &= 1+5t \\ y &= 2\sqrt{3}-2t \end{aligned}\right\} t\in\mathbb{R} \Rightarrow p = \{[1+5t;\, 2\sqrt{3}-2t];\ t\in\mathbb{R}\}$$

$$P: \left.\begin{aligned} 2 &= 1+5t \Rightarrow t = \tfrac{1}{5} \\ \sqrt{3} &= 2\sqrt{3}-2t \Rightarrow t = \tfrac{\sqrt{3}}{2} \end{aligned}\right\} P\notin p$$

→ vzájemná poloha přímek v π_2

- $p \parallel q \iff \vec{v}_p = k\cdot\vec{v}_q$ (1)
- $p \nparallel q \iff \vec{v}_p \neq k\cdot\vec{v}_q$ (2)
- $p \equiv q \iff \vec{v}_p = k\cdot\vec{v}_q \ \wedge\ \vec{AB} = l\cdot\vec{v}_q$ (3)





→ **příklad** ⇒ jsou totožné ?

• $p = \{[2-t;\ 3+4t];\ t \in \mathbb{R}\} \wedge q: Q[3;-4] \wedge p \parallel q \Rightarrow \vec{u}_q = ?$

→ $p \parallel q \Rightarrow u_q = k \cdot u_p \Rightarrow \vec{u}_q = (-1;4)$

→ **totožnost**

a) $\left.\begin{array}{l} 3 = 2 - t \Rightarrow t = -1 \\ -4 = 3 + 4t \Rightarrow t = -\frac{7}{4} \end{array}\right\} Q \notin p \Rightarrow p \not\equiv q$

b) $P[2;3] \Rightarrow \vec{PQ} = (1;-7) \wedge \vec{PQ} = k \cdot \vec{u}_q$

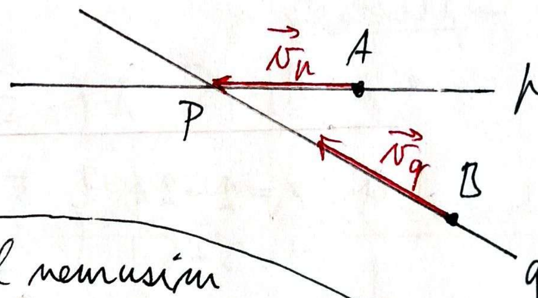
$\left.\begin{array}{l} \Rightarrow 1 = -k \Rightarrow k = -1 \\ -7 = 4k \Rightarrow k = -\frac{7}{4} \end{array}\right\} p \not\equiv q$

→ **průsečík různoběžek**

$p: X_p = [3;1] + t(1;2);\ t \in \mathbb{R}$

$q: X_q = [2;3] + t(-1;4);\ t \in \mathbb{R}$

$\left.\begin{array}{l} 1 = -k \Rightarrow k = -1 \\ 2 = 4k \Rightarrow k = \frac{1}{2} \end{array}\right\} p \nparallel q$



→ $P[x;y] = ?$

$x = 3 + l \wedge x = 2 - k$

$y = 1 + 2l \wedge y = 3 + 4k$

l nemusím dopočítávat

$3 + l = 2 - k \Rightarrow l = -1 - k$

$1 + 2l = 3 + 4k \Rightarrow 1 - 2 - 2k = 3 + 4k \Rightarrow -4 = 6k \Rightarrow \underline{\underline{k = -\frac{2}{3}}}$

$\left.\begin{array}{l} \rightarrow x = 2 - k = \frac{8}{3} \\ \rightarrow y = 3 + 4k = \frac{1}{3} \end{array}\right\} \underline{\underline{P\left[\frac{8}{3};\ \frac{1}{3}\right]}}$

→ Obecná rovnice přímky – z lin. fce

$a \cdot x + b \cdot y + c = 0$; $a, b, c \in R$ ∧ alespoň jedno z $a, b \neq 0$

→ převod z parametrické na obecnou rovnici

$p: A[a_1; a_2], \vec{v}_p(v_1; v_2)$ – zbavím se parametru

$p: x = a_1 + t \cdot v_1 \quad | \cdot v_2$

$y = a_2 + t \cdot v_2 \quad | \cdot (-v_1)$

$\left.\begin{array}{l} x v_2 = a_1 \cdot v_2 + t v_1 \cdot v_2 \\ -y v_1 = -a_2 \cdot v_1 - t v_1 \cdot v_2 \end{array}\right\} \oplus$

$v_2 \cdot x - v_1 \cdot y = v_2 \cdot a_1 - v_1 \cdot a_2$

$c = 0 \Leftrightarrow A = [0; 0]$

$v_2 \cdot x - v_1 \cdot y - v_2 \cdot a_1 + v_1 \cdot a_2 = 0 \sim ax + by + c = 0$

$\vec{v}_p(v_1; v_2) \wedge \vec{n}_p = (v_2; -v_1) = (a; b)$

↳ normálový vektor $\Rightarrow \vec{v}_p \perp \vec{n}_p$

→ příklady

- $p: A[2; 3], \vec{v}(-2; 5) \rightarrow$ vyjádři obecnou rovnicí

a) $\left.\begin{array}{l} x = 2 - 2t \\ y = 3 + 5t \end{array}\right\}$ $\begin{array}{l} 5x = 10 - 10t \\ 2y = 6 + 10t \end{array}$

$5x + 2y = 16 \Rightarrow 5x + 2y - 16 = 0$

b) $\vec{v}(-2; 5) \Rightarrow \vec{n}(5; 2) \wedge A[2; 3]$

$5x + 2y + c = 0 \Rightarrow 5 \cdot 2 + 2 \cdot 3 = -c \Rightarrow c = 16$

- $p: A[3; -2] \wedge q: 2x - y = 0 \wedge p \perp q \Rightarrow p = ?$

$\vec{v}_p \perp \vec{v}_q \Rightarrow \vec{v}_p = \vec{n}_q = (2; -1) \Rightarrow \vec{n}_p = (1; 2)$

a) $p = \{[3 + 2t; -2 - t]; t \in \mathbb{R}\}$ A

b) $p: x + 2y + c = 0 \Rightarrow 3 - 4 = -c \Rightarrow c = 1 \Rightarrow p: x + 2y + 1 = 0$

→ Vzájemná poloha přímek obecnou rovnicí

$p: ax + by + c = 0$

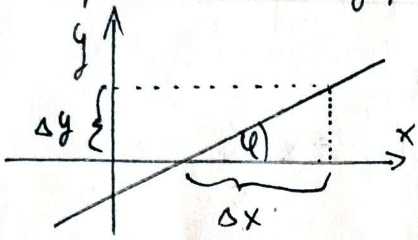
$q \parallel p: ax + by + d = 0 \quad \wedge \quad q \equiv p \Leftrightarrow c = d$

$q \nparallel p: ex + fy + d = 0 \quad \wedge \quad e \neq a \vee f \neq b \vee e, f \neq a, b$

→ Směrová rovnice – z lin. fce

$y = kx + q \wedge k \neq 0$

- k = směrnice → $k = tg(\varphi)$ – φ = směrový úhel
- q = úsek který přímka vytne na ose y



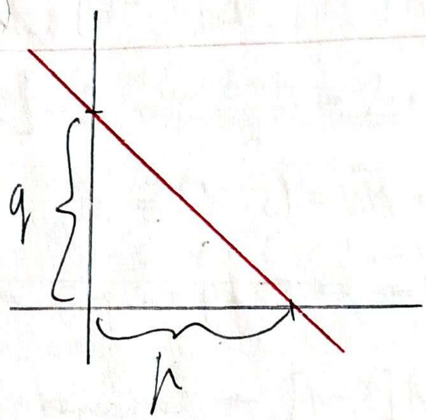
$\to k = \frac{\Delta y}{\Delta x}$

$tg\varphi = \frac{\Delta y}{\Delta x}$ $\Big\}$ $k = tg\varphi$

→ Úseková rovnice

$\frac{x}{p} + \frac{y}{q} = 1$ $p, q \neq 0$

- p = úsek na ose x
- q = úsek na ose y

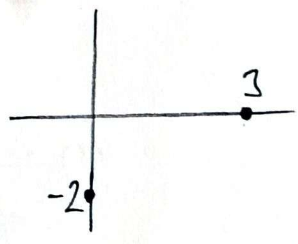


→ převod na obecnou rovnici

$x\cdot q + y\cdot p = p\cdot q \Rightarrow x\cdot q + y\cdot p - p\cdot q = 0$

→ příklady

- $p = \leftrightarrow AB \wedge A[3;0] \wedge B[0;-2]$ → vyjádři p všemi způsoby
  - $\frac{x}{3} - \frac{y}{2} = 1$ $\cdot 6$
  - $2x - 3y - 6 = 0$
  - $3y = 2x - 6 \Rightarrow y = \frac{2}{3}x - 2$
  - $\vec{n}(2;-3) \Rightarrow \vec{v}(3;2) \Rightarrow p = \{[3+3t; 2t]; t \in \mathbb{R}\}$



2) $p: 2x + 5y - 6 = 0$ → parametrické a směrnicové vyjádření

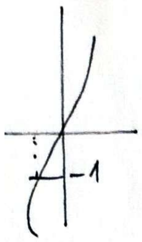
a) $\vec{n} = (2;5) \Rightarrow \vec{v} = (5;-2)$

$y = 0: 2x = 6$, $x = 3$ $\Big\}$ $A = [3;0]$

$x = 3 + 5t$

$y = 0 - 2t$, $t \in \mathbb{R}$

b) $5y = -2x + 6 \Rightarrow y = -\frac{2}{5}x + \frac{6}{5}$



3) $p: A[0;2], B[-2;4] \Rightarrow tg(\varphi),\ \varphi = ?$

$$tg\,\varphi = k = \frac{\Delta y}{\Delta x} = \frac{2}{-2} = \underline{\underline{-1}} \Rightarrow \varphi = -\frac{\pi}{4}\ (+\pi) \Rightarrow \underline{\underline{\varphi = \frac{3}{4}\pi}}$$

4) $q: 4x - y + 3 = 0;\ p: O[0;0];\ p \parallel q \rightarrow p$ parametricky

$$\left.\begin{aligned} \vec{n} = (4;-1) \Rightarrow \vec{u} &= (1;4) \\ A &= [0;0] \end{aligned}\right\} \begin{aligned} x &= t \\ y &= 4t;\ t \in \mathbb{R} \end{aligned}$$

10) $A[2;4], B[4;-6], M[-4;-3], N[1;-2] \Rightarrow S_{AB} \in p \wedge p \perp \overleftrightarrow{MN}$

- $S = \frac{A+B}{2} \Rightarrow S = [3;-1]$
- $\vec{MN} = (5;1) = \vec{n}_p$

$\Rightarrow p: 5x + y + c = 0 \Rightarrow S \in p: 15 - 1 = -c \Rightarrow \underline{\underline{p: x - 5y - 14 = 0}}$

11) $A[3;-1] \rightarrow$ parametrické vyjádření + obecná rce. $p = ?$

a) $q: 2x + 3y + 7 = 0 \wedge p \parallel q$

$\Rightarrow p: 2x + 3y + c = 0 \Rightarrow A: 6 - 3 = -c \Rightarrow \underline{\underline{p: 2x + 3y - 3 = 0}}$

$\rightarrow \vec{n}_p = (2;3) \Rightarrow \vec{u}_p = (3;-2) \Rightarrow p: \left.\begin{aligned} x &= 3 + 3t \\ y &= -1 - 2t \end{aligned}\right\} t \in \mathbb{R}$

b) $q: x - 2y + 4 = 0 \wedge p \perp q \Rightarrow \vec{n}_q = (1;-2) = \vec{u}_p \Rightarrow \vec{n}_p = (2;1)$

$\Rightarrow p: 2x + y + c = 0 \Rightarrow A: 6 - 1 = -c \Rightarrow \underline{\underline{p: 2x + y - 5 = 0}}$

$\rightarrow \vec{u}_p = (1;-2) \Rightarrow p: \left.\begin{aligned} x &= 3 + t \\ y &= -1 - 2t \end{aligned}\right\} t \in \mathbb{R}$

c) $p \parallel \vec{x} \Rightarrow \vec{u}_p = (x;0) \rightarrow$ např: $\vec{u}_p = (1;0) \Rightarrow \vec{n}_p = (0;1)$

$\Rightarrow p: y + c = 0 \Rightarrow A: -1 = -c \Rightarrow \underline{\underline{p: y + 1 = 0}}$

$\rightarrow \vec{u}_p = (1;0) \Rightarrow p: \left.\begin{aligned} x &= 3 + t \\ y &= -1 \end{aligned}\right\} t \in \mathbb{R}$

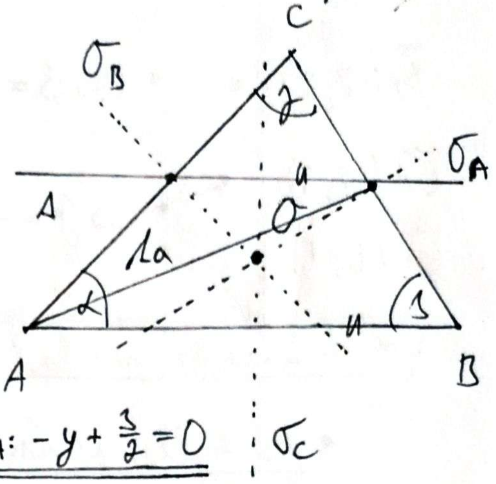
d) $p \parallel \vec{y} \Rightarrow \vec{u}_p = (0;y) \rightarrow$ např: $\vec{u}_p = (0;1) \Rightarrow \vec{n}_p = (1;0)$

$\Rightarrow p: x + c = 0 \Rightarrow A: 3 = -c \Rightarrow \underline{\underline{p: x - 3 = 0}}$

$\rightarrow \vec{u}_p = (0;1) \Rightarrow p: \left.\begin{aligned} x &= 3 \\ y &= -1 + t \end{aligned}\right\} t \in \mathbb{R}$

14) $A[2;4]$, $B[4;2]$, $C[4;1]$ $\rightarrow \triangle ABC$

a) obecné rovnice os stran + jejich průsečík = s. k. opsané



- $S_{BC}\left[4; \frac{3}{2}\right]$ $S_{AC}\left[3; \frac{5}{2}\right]$ $S_{AB}[3;3]$
- $\vec{AB}(2;-2)$ $\vec{AC}(2;-3)$ $\vec{BC}(0;-1)$
- $o_A \perp \vec{BC} \wedge S_{BC} \in o_A$

$\vec{n}_{oA} = \vec{BC} = (0;-1)$

$o_A: -y + c = 0 \rightarrow S_{BC}: -\frac{3}{2} = -c \rightarrow o_A: -y + \frac{3}{2} = 0$

- $o_B: \vec{n}_{o_B} = \vec{AC} = (2;-3) \Rightarrow o_B: 2x - 3y + c = 0$

$S_{AC} \in o_B: 6 - \frac{15}{2} = -c \Rightarrow -\frac{3}{2} = -c \Rightarrow o_B: 2x - 3y + \frac{3}{2} = 0$

- $o_C: \vec{n}_{o_C} = \vec{AB} = (2;-2) \Rightarrow o_C: 2x - 2y + c = 0$

$S_{AB} \in o_C: 6 - 6 = -c \Rightarrow o_C: 2x - 2y = 0$

- $O \in o_A \wedge O \in o_C \rightarrow O[o_1; o_2]$

$$\left.\begin{array}{l} o_A: y = \frac{3}{2} \Rightarrow o_2 = \frac{3}{2} \\ o_C: y = x \Rightarrow o_2 = o_1 \Rightarrow o_1 = \frac{3}{2} \end{array}\right\} O\left[\frac{3}{2}; \frac{3}{2}\right]$$

b) $s: s \parallel AB \wedge S_{AC} \in s \wedge S_{BC} \in s$

$$\vec{v}_s = \vec{AB} = (2;-2) \rightarrow s: \left.\begin{array}{l} x = 4 + 2t \\ y = \frac{3}{2} - 2t \end{array}\right\} t \in \langle 0;1 \rangle$$

$\rightarrow |S_{AC} S_{BC}| = \sqrt{1^2 + 1^2} = \sqrt{2}$

c) poloměr kružnice opsané: $r = |OA| \rightarrow \vec{OA} = \left(\frac{1}{2}; \frac{5}{2}\right)$

$r = |\vec{OA}| = \sqrt{\frac{1}{4} + \frac{25}{4}} = \frac{\sqrt{26}}{2}$

d) délka $t_a = |A S_{BC}| \rightarrow \vec{AS}_{BC} = \left(2; -\frac{5}{2}\right)$

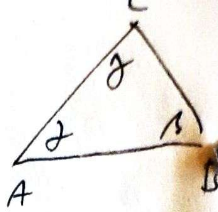
$t_a = \sqrt{\frac{16}{4} + \frac{25}{4}} = \frac{\sqrt{41}}{2}$

e) obsah: $S = \frac{1}{2}|\vec{AB} \times \vec{AC}| = \frac{1}{2}|\vec{w}|$

$$\left.\begin{array}{l} \vec{AB} = (2;-2;0)\ 2;-2 \\ \vec{AC} = (2;-3;0)\ 2;-3 \end{array}\right\} \begin{array}{l} w_1 = 0 \\ w_2 = 0 \\ w_3 = -6 + 4 = -2 \end{array}$$

$\Rightarrow \frac{1}{2}|\vec{w}| = \frac{1}{2}\sqrt{0 + 0 + 4} = 1\,j^2$

f) $\alpha, \beta, \gamma = ?$ → $\vec{AB}(2;-2)$ $\vec{AC}(2;-3)$ $\vec{BC}(0;-1)$



- $\cos\alpha = \dfrac{\vec{AB}\cdot\vec{AC}}{|\vec{AB}|\cdot|\vec{AC}|} = \dfrac{4+6}{\sqrt{8}\cdot\sqrt{13}} = \dfrac{10}{2\sqrt{26}} \Rightarrow \underline{\underline{\alpha \doteq 11{,}3^\circ}}$

$\vec{BA}(-2;2)$ ← • $\cos\beta = \dfrac{\vec{BA}\cdot\vec{BC}}{|\vec{BA}|\cdot|\vec{BC}|} = \dfrac{0-2}{\sqrt{8}\cdot 1} = \dfrac{-2}{2\sqrt{2}} = -\dfrac{\sqrt{2}}{2} \Rightarrow \underline{\underline{\beta = 135^\circ}}$

$\vec{CA}(-2;3)$ ← • $\cos\gamma = \dfrac{\vec{CA}\cdot\vec{CB}}{|\vec{CA}|\cdot|\vec{CB}|} = \dfrac{0+3}{\sqrt{13}\cdot 1} = \dfrac{3\sqrt{13}}{13} \Rightarrow \underline{\gamma \doteq 33{,}7^\circ}$

$\vec{CB}(0;1)$

→ Metrické vztahy přímek v E_2

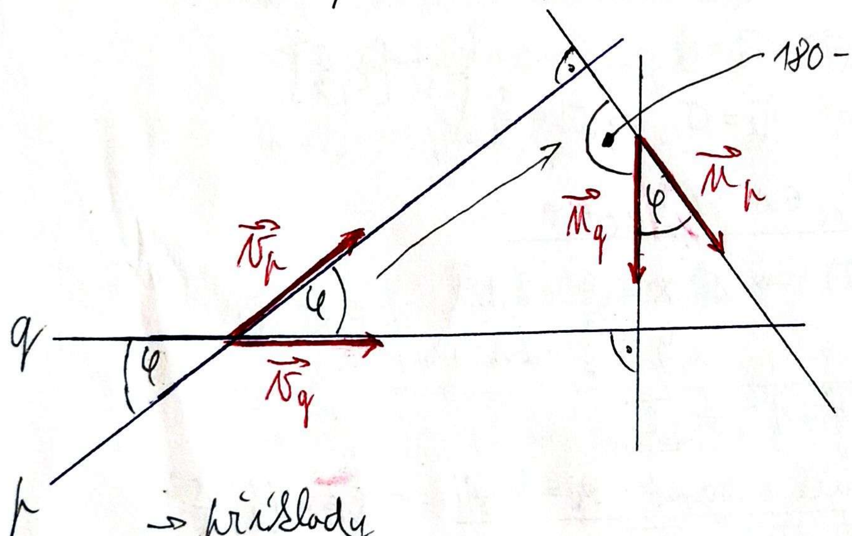
- Odchylka přímek

→ menší z úhlů, které ty přímky svírají

→ cos je sudá fce ⇒ absolutní hodnota

$$\cos(\varphi) = \left|\frac{\vec{v}_p\cdot\vec{v}_q}{|\vec{v}_p|\cdot|\vec{v}_q|}\right| = \left|\frac{\vec{n}_p\cdot\vec{n}_q}{|\vec{n}_p|\cdot|\vec{n}_q|}\right|$$

→ můžu použít i normálové vektory



→ příklady

- $p = \{[2+t;5];\ t\in\mathbb{R}\} \wedge q: x + \sqrt{3}\cdot y - 6 = 0$

$\vec{v}_p = (1;0) \wedge \vec{n}_q = (1;\sqrt{3}) \Rightarrow \vec{v}_q = (\sqrt{3};-1)$

$\cos\varphi = \dfrac{\sqrt{3}+0}{\sqrt{1+3}\cdot\sqrt{0+1}} = \dfrac{\sqrt{3}}{2} \Rightarrow \underline{\underline{\varphi = 30^\circ}}$

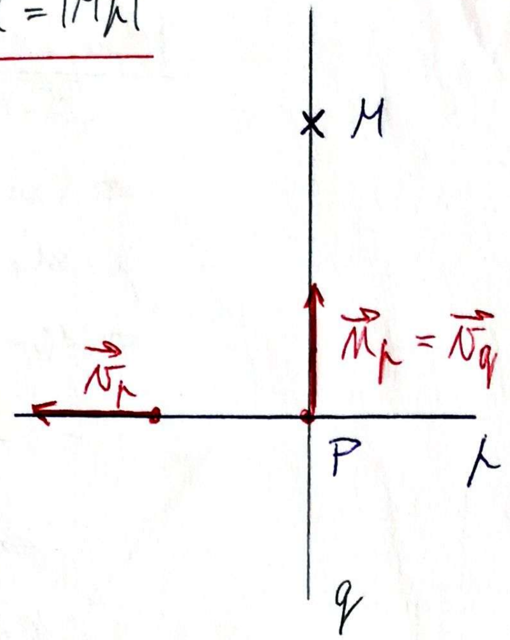
- $p: x+2y-1=0 \wedge q: 2x-y+4=0$

$\vec{n}_p = (1;2) \wedge \vec{n}_q = (2;-1)$

$\cos\varphi = \dfrac{2-2}{\sqrt{5}\cdot\sqrt{5}} = 0 \Rightarrow \underline{\underline{\varphi = 90^\circ}}$

• Vzdálenost bodu od přímky

$\Rightarrow M[m_1; m_2] \wedge p: ax + by + c = 0 \Rightarrow d = |Mp|$



- $\vec{n}_p = \vec{v}_q = (a; b) \Rightarrow q: X = M + t \cdot \vec{v}_q$
- $q: \left.\begin{aligned} x &= m_1 + a \cdot t \\ y &= m_2 + b \cdot t \end{aligned}\right\} t \in \mathbb{R}$
- $P: a(m_1 + a \cdot t) + b(m_2 + b \cdot t) + c = 0$

$$a \cdot m_1 + a^2 \cdot t + b \cdot m_2 + b^2 \cdot t + c = 0$$

$$t(a^2 + b^2) = -a \cdot m_1 - b \cdot m_2 - c$$

$$t = -\frac{a \cdot m_1 + b \cdot m_2 + c}{a^2 + b^2}$$

$$\Rightarrow p_1 = m_1 + a \cdot t = m_1 - a\frac{a \cdot m_1 + b \cdot m_2 + c}{a^2 + b^2}$$

$$p_2 = m_2 + b \cdot t = m_2 - b\frac{a \cdot m_1 + b \cdot m_2 + c}{a^2 + b^2}$$

- $d = |MP| = \sqrt{(m_1 - p_1)^2 + (m_2 - p_2)^2} = \sqrt{(-a \cdot t)^2 + (-b \cdot t)^2} = \sqrt{(-t)^2(a^2 + b^2)}$

$$d = \sqrt{\frac{(a \cdot m_1 + b \cdot m_2 + c)^2 (a^2 + b^2)}{(a^2 + b^2)^2}} = \frac{\sqrt{(a \cdot m_1 + b \cdot m_2 + c)^2}}{\sqrt{a^2 + b^2}} \quad \wedge \sqrt{x^2} = |x|$$

$$\Rightarrow d = \frac{|a \cdot m_1 + b \cdot m_2 + c|}{\sqrt{a^2 + b^2}}$$

$\Rightarrow$ příklady

- $A[8; -5] \wedge p = \{[-4t; \frac{7}{2} + 3t], t \in \mathbb{R}\}$

$p: P[0; \frac{7}{2}] \wedge \vec{v}_p = (-4; 3) \Rightarrow \vec{n}_p = (3; 4)$

$p: 3x + 4y + c = 0 \Rightarrow P \in p: 14 + c = 0 \Rightarrow p: 3x + 4y - 14 = 0$

$$d = \frac{|24 - 20 - 14|}{\sqrt{9 + 16}} = \frac{10}{5} = 2$$

- $p: x+3y-2=0 \;\wedge\; q: 5x+12y-4=0 \;\wedge\; v(M;q)=3 \rightarrow M\in p = ?$

$$\frac{|5\cdot m_1+12\cdot m_2-4|}{\sqrt{25+144}}=3 \Rightarrow |5m_1+12m_2-4|=39$$

$$\Rightarrow 5m_1+12m_2-4=\pm 39$$

$$p: m_1+3m_2-2=0 \Rightarrow m_1=2-3m_2 = 2-6\pm 39 \begin{cases} 35 \\ -43 \end{cases}$$

$$\Rightarrow 10-15m_2+12m_2=4\pm 39$$

$$-3m_2=-6\pm 39 \Rightarrow m_2=2\mp 13 \begin{cases} -11 \\ 15 \end{cases}$$

$$\Rightarrow M_1[35;-11] \quad M_2[-43;15]$$

- $p: 8x-6y+3=0 \;\wedge\; q: 8x-6y-3=0 \;\wedge\; p\parallel q \Rightarrow v(p;q)=?$

→ najdu bod na p a pak jeho vzdálenost od q

$$\Rightarrow P: x=0 \Rightarrow -6y+3=0 \Rightarrow y=\frac{1}{2} \Rightarrow P\left[0;\frac{1}{2}\right]$$

$$\Rightarrow v(P;q)=v(p;q)=\frac{|0-3-3|}{\sqrt{64+36}}=\frac{6}{10}=\underline{\underline{\frac{3}{5}}}$$

→ osa rovinného pásu

$$\left.\begin{aligned} p_1&: ax+by+c_1=0 \\ p_2&: ax+by+c_2=0 \end{aligned}\right\} \sigma: ax+by+\frac{c_1+c_2}{2}=0$$

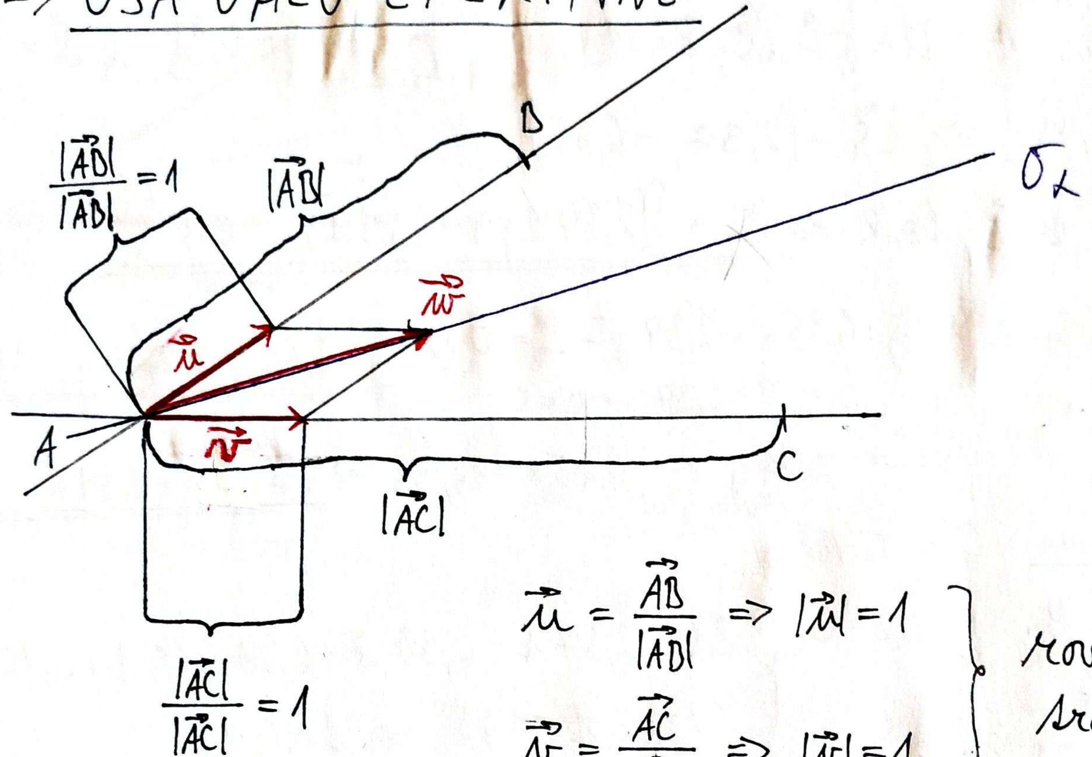
→ směrnice kolmice

$$\left.\begin{aligned} &p: ax+by+c=0 \rightarrow \vec{n}(a;b) \Rightarrow \vec{v}(-b;a) \\ &y=-\frac{a}{b}x-\frac{c}{b} \Rightarrow k_p=-\frac{a}{b} \end{aligned}\right\} \underline{k=\frac{v_2}{v_1}}$$

$$q\perp p: -bx+ay+d=0 \Rightarrow \vec{n}(-b;a) \rightarrow \vec{v}(a;b) \Rightarrow k_q=\frac{b}{a}$$

$$\Rightarrow \underline{k_p\cdot k_q=-1} \iff \underline{k_q=-\frac{1}{k_p}}$$

⇒ OSA ÚHLU EFEKTIVNĚ



$$\left.\begin{aligned} \vec{u} &= \frac{\vec{AB}}{|\vec{AB}|} \Rightarrow |\vec{u}| = 1 \\ \vec{v} &= \frac{\vec{AC}}{|\vec{AC}|} \Rightarrow |\vec{v}| = 1 \end{aligned}\right\}$$ rovnoramenný trojúhelník

⇒ $\underline{\vec{w} = \vec{u} + \vec{v}}$ – směr. vektor osy

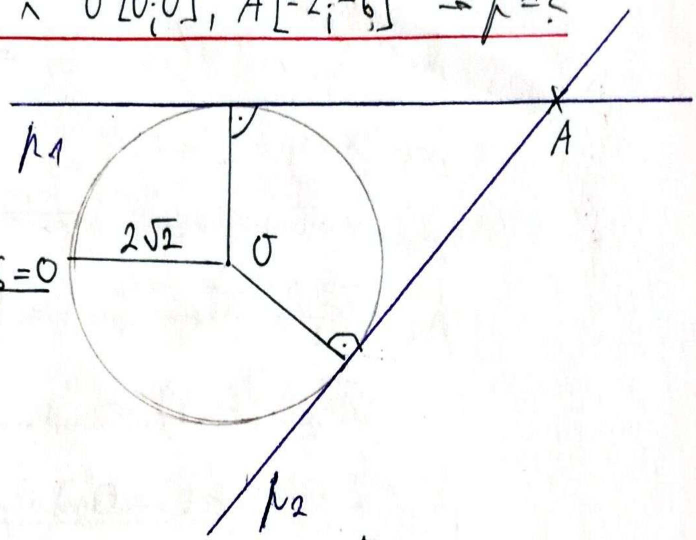
⇒ příklad: $A[1;2],\ B[-1;0],\ C[3;-2]$ ⇒ $\vec{AB} = (-2;-2)$, $\vec{AC} = (2;-4)$

$|\vec{AB}| = \sqrt{8} = 2\sqrt{2} \ \wedge \ |\vec{AC}| = \sqrt{20} = 2\sqrt{5}$

$$\left.\begin{aligned} \Rightarrow \vec{u} &= \left(-\frac{2}{2\sqrt{2}}; -\frac{2}{2\sqrt{2}}\right) = \left(-\frac{\sqrt{2}}{2}; -\frac{\sqrt{2}}{2}\right) \\ \vec{v} &= \left(\frac{2}{2\sqrt{5}}; -\frac{4}{2\sqrt{5}}\right) = \left(\frac{\sqrt{5}}{5}; -\frac{2\sqrt{5}}{5}\right) \end{aligned}\right\} \underline{\underline{\vec{w} = \left(\frac{\sqrt{5}}{5} - \frac{\sqrt{2}}{2}; -\frac{2\sqrt{5}}{5} - \frac{\sqrt{2}}{2}\right)}}$$

→ príklady

• $p: A \in p \ \wedge\ v(O;p) = 2\sqrt{2} \ \wedge\ O[0;0],\ A[-2;-6] \ \rightarrow\ p = ?$



$p: y = kx + q$

$A \in p: -6 = -2k + q \Rightarrow q = 2k - 6$

$p: y = kx + 2k - 6 \Rightarrow \underline{kx - y + 2k - 6 = 0}$

$v(O;p): \dfrac{|0 - 0 + 2k - 6|}{\sqrt{k^2+1}} = \dfrac{2|k-3|}{\sqrt{k^2+1}} = 2\sqrt{2}$

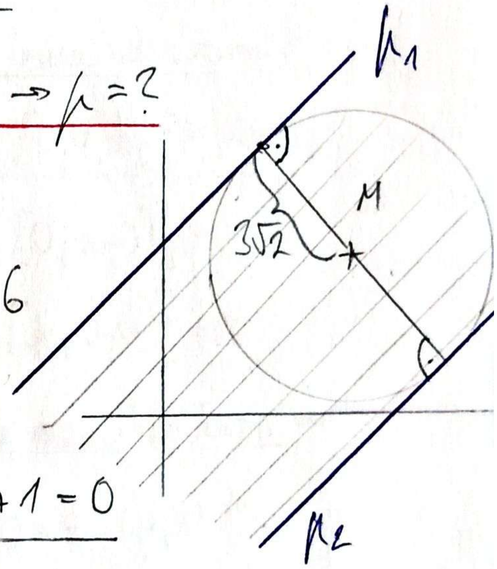
$\Rightarrow |k-3| = \sqrt{2}\cdot\sqrt{k^2+1}$

$k^2 - 6k + 9 = 2k^2 + 2 \Rightarrow \underline{k^2 + 6k - 7 = 0} \ \begin{cases} -7 \\ 1 \end{cases}$

$\Rightarrow p_1: y = -7x - 14 - 6 \Rightarrow \underline{\underline{p_1: 7x + y + 20 = 0}}$

$\Rightarrow p_2: y = x + 2 - 6 \Rightarrow \underline{\underline{p_2: x - y - 4 = 0}}$

• $k_p = 1 \ \wedge\ v(M;p) = 3\sqrt{2} \ \wedge\ M[-5;2] \ \rightarrow\ p = ?$



$p: y = x + c \Rightarrow x - y + c = 0$

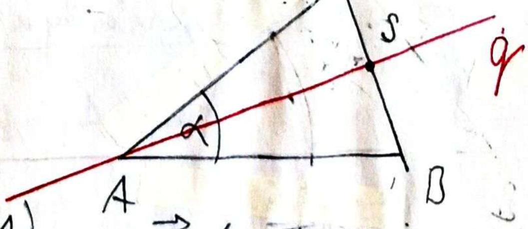
$v(M;p): \dfrac{|-5 - 2 + c|}{\sqrt{1+1}} = 3\sqrt{2} \Rightarrow |c - 7| = 6$

$\Rightarrow c - 7 = \pm 6 \Rightarrow c = 7 \pm 6 \ \begin{cases} 13 \\ 1 \end{cases}$

$\Rightarrow \underline{p_1: x - y + 13 = 0} \ \wedge\ \underline{p_2: x - y + 1 = 0}$

• $A[2;-4],\ B[1;-2],\ C[0;-3] \Rightarrow \triangle ABC \ \wedge\ q = O(\alpha) = ?$

$\left.\begin{array}{l} \vec{AB} = (-1;2) \Rightarrow |\vec{AB}| = \sqrt{5} \\ \vec{AC} = (-2;1) \Rightarrow |\vec{AC}| = \sqrt{5} \end{array}\right\}$ rovnoramenný $\triangle$



$\Rightarrow S = \dfrac{B + C}{2} = \left[\dfrac{1}{2}; -\dfrac{5}{2}\right]$

$\Rightarrow \vec{u}_q = \vec{AS} = \left(-\dfrac{3}{2}; \dfrac{3}{2}\right) = (-3;3) = (-1;1) \Rightarrow \vec{n}_q(1;1)$

$\left.\begin{array}{l} \Rightarrow q: x + y + c = 0 \\ \Rightarrow A \in q: 2 - 4 + c = 0 \end{array}\right\} \ \underline{\underline{q: x + y + 2 = 0}}$

• $M[4;6]$, $A[-6;10]$, $B[10;-6]$, $p: M \in p \wedge v(A;p) = v(B;p) \Rightarrow p = ?$

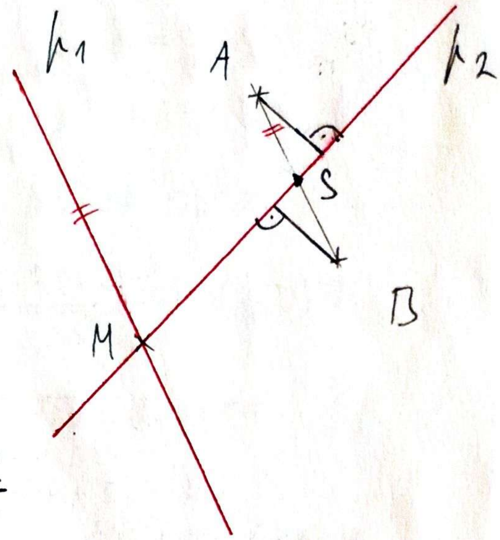
$p_1: \vec{v}_1 = \vec{AB} = (16;-16) \Rightarrow \vec{n}_1(1;1)$

$$\left.\begin{array}{l} p_1: x+y+c=0 \\ M\in p_1: 4+6+c=0 \end{array}\right\} \underline{\underline{p_1: x+y-10=0}}$$

$p_2: \vec{v}_2 = \vec{SM} \wedge S=[2;2]$

$\vec{v}_2 = (2;4) \Rightarrow n_2(2;-1)$

$$\left.\begin{array}{l} p_2: 2x-y+c=0 \\ M\in p_2: 8-6+c=0 \end{array}\right\} \underline{\underline{p_2: 2x-y-2=0}}$$



→ Přímka v prostoru

$p: X = A + t\cdot\vec{v};\ t\in\mathbb{R}$ – jen parametrické vyjádření

→ souřadnicové osy

$x = \{[t;0;0];\ t\in\mathbb{R}\}$ → $O[0;0;0]\in x \wedge \vec{v}_x(1;0;0)$

$y = \{[0;t;0];\ t\in\mathbb{R}\}$

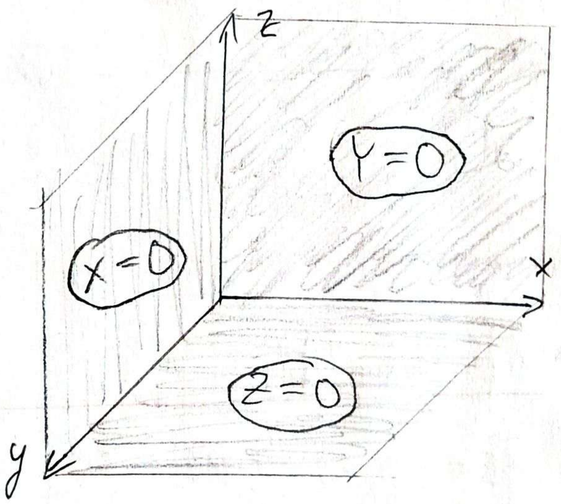
$z = \{[0;0;t];\ t\in\mathbb{R}\}$

→ souřadnicové roviny

$\rho(x,y): z=0$

$\sigma(y,z): x=0$

$\tau(x,z): y=0$



→ průsečík přímky a roviny

$p: \{[2;\ 1-t;\ 4t];\ t\in\mathbb{R}\}$ → průsečíky se souřadnicovými rovinami

$$\left.\begin{array}{l} \rho(x;y): 4t=0 \Rightarrow \underline{t=0} \Rightarrow \underline{\underline{P_\rho[2;1;0]}} \\ \sigma(y;z): 2\neq 0 \Rightarrow P_\sigma \text{ neexistuje} \Rightarrow \underline{\underline{p\parallel\sigma}} \\ \tau(x;z): 1-t=0 \Rightarrow \underline{t=1} \Rightarrow P_\tau[2;0;4] \end{array}\right\} p: X = P_\rho + t(P_\tau - P_\rho)$$

→ Vzájemná poloha přímek v prostoru

$p: X = A + t\cdot\vec{v}\ ;\ t\in\mathbb{R}$

$q: X = B + s\cdot\vec{u}\ ;\ s\in\mathbb{R}$

- $\vec{v} = k\cdot\vec{u} \wedge \vec{AB} = l\cdot\vec{u}\ (A\in q) \iff p \equiv q$
- $\vec{v} = k\cdot\vec{u} \wedge \vec{AB} \neq l\cdot\vec{u}\ (A\notin q) \iff p \parallel q$
- $\vec{v} \neq k\cdot\vec{u} \wedge \vec{AB} = k\cdot\vec{v} + l\cdot\vec{u} \iff p \nparallel q$

→ $p, q, \leftrightarrow AB$ jsou komplanární

$\Rightarrow$ dají se umístit do 1 roviny

- $\vec{v} \neq k\cdot\vec{u} \wedge \vec{AB} \neq k\cdot\vec{v} + l\cdot\vec{u} \iff p \not\asymp q$ – mimoběžky

→ $p, q, \leftrightarrow AB$ nejsou komplementární

→ příklady

- $p = \{[2+k; 3-2k; 4]; k\in\mathbb{R}\},\ q = \{[1-4t; m+t; 1-3t]; t\in\mathbb{R}\}$

$p \nparallel q \Rightarrow m = ?$

$\Rightarrow P[2;3;4] \wedge Q[1;m;1] \Rightarrow \vec{QP} = (1; 3-m; 3)$

$\Rightarrow \vec{v}_p(1;-2;0) \wedge \vec{v}_q(-4;1;-3) \Rightarrow \vec{v}_p \neq k\cdot\vec{v}_q$

$\Rightarrow \vec{QP} = k\cdot\vec{v}_p + l\cdot\vec{v}_q$

$1 = k - 4l \Rightarrow k = -3$

$3 - m = -2k + l \Rightarrow m = 3 - 6 + 1 = \underline{\underline{-2}}$

$3 = -3l \Rightarrow l = -1$

- $p = \{[-3+2t; -1+2t; 4t]; t\in\mathbb{R}\},\ q = \{[3+s; -1+2s; 4]; s\in\mathbb{R}\}$

$\Rightarrow P[-3;-1;0] \wedge Q[3;-1;4] \Rightarrow \vec{PQ} = (6;0;4)$

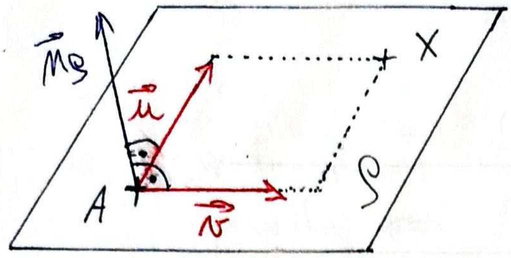
$\Rightarrow \vec{v}_p(2;2;4) \wedge \vec{v}_q(1;2;0) \Rightarrow \vec{v}_p \neq k\cdot\vec{v}_q \Rightarrow p \nparallel q \vee p \not\asymp q$

$\Rightarrow \vec{PQ} = k\cdot\vec{v}_p + l\cdot\vec{v}_q$

$$\left.\begin{array}{l} 6 = 2k + l \Rightarrow 6 \neq 2 - 1 \\ 0 = 2k + 2l \Rightarrow l = -1 \\ 4 = 4k \Rightarrow k = 1 \end{array}\right\} \underline{\underline{p \not\asymp q}}$$

→ Rovina v prostoru

• parametrické vyjádření



$\vec{u} \neq k\cdot\vec{v} \wedge \vec{u}, \vec{v} \neq \vec{o}(0;0;0)$

$\vec{AX} = (X-A) = t\cdot\vec{v} + l\cdot\vec{u}$

$\Rightarrow \rho: X = A + t\cdot\vec{v} + l\cdot\vec{u};\ t, l \in \mathbb{R}$

• obecná rovnice

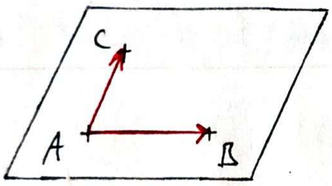
$\rho: ax + by + cz + d = 0$

$\Rightarrow \vec{n}_\rho = (a;b;c) \neq (0;0;0)$

$\Rightarrow \vec{n}_\rho \perp \vec{u} \wedge \vec{n}_\rho \perp \vec{v} \Rightarrow \vec{n}_\rho = \vec{u} \times \vec{v}$

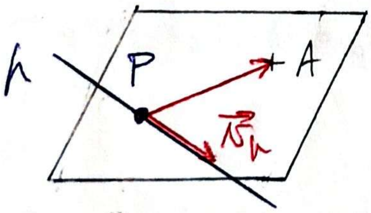
→ určení roviny

• $\rho = \leftrightarrow ABC$ – A, B, C – nekolineární



$\rho: X = A + t\cdot\vec{AB} + l\cdot\vec{AC};\ t, l \in \mathbb{R}$

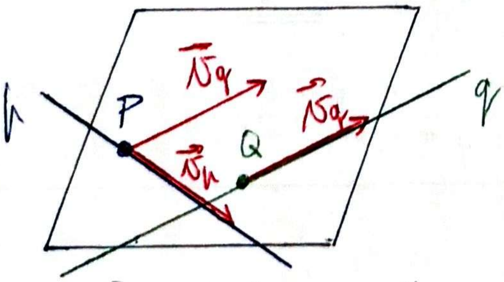
• $\rho = \leftrightarrow pA$ – $A \notin p \wedge p: X = P + t\cdot\vec{v}_p$



$\rho: X = P + t\cdot\vec{v}_p + l\cdot\vec{PA};\ t, l \in \mathbb{R}$

• $\rho = \leftrightarrow pq$ – $p \nparallel q \wedge p: X = P + t\cdot\vec{v}_p$

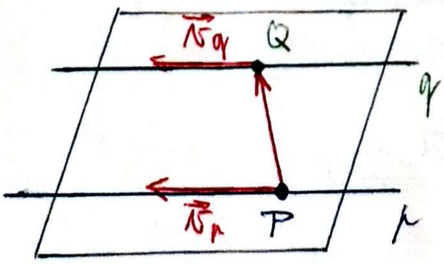
$q: X = Q + l\cdot\vec{v}_q$



$\rho: X = P + t\cdot\vec{v}_p + l\cdot\vec{v}_q;\ t, l \in \mathbb{R}$

• $\rho = \leftrightarrow pq$ – $p \parallel q \wedge p \not\equiv q \wedge p: X = P + t\cdot\vec{v}_p$

$q: X = Q + t\cdot\vec{v}_q$



$\rho: X = P + t\cdot\vec{v}_p + l\cdot\vec{PQ};\ t, l \in \mathbb{R}$

→ příklady

- $A[2;1;6],\ B[0;-1;-6],\ C[-1;2;0] \rightarrow \leftrightarrow ABC = \rho = ?$

$\rho: X = A + k\cdot\vec{AB} + l\cdot\vec{AC};\ k,l \in \mathbb{R}$

$\Rightarrow \vec{AB}(-2;-2;-12) \wedge \vec{AC}(-3;1;-6) \Rightarrow \vec{AB} \neq k\cdot\vec{AC} \Rightarrow A,B,C$ - nekolin.

$$\Rightarrow \rho: \left.\begin{aligned} x &= 2-2k-3l \\ y &= 1-2k+l \\ z &= 6-12k-6l \end{aligned}\right\} k,l \in \mathbb{R}$$

$\Rightarrow$ průsečíky s osami

$P_x: y=0 \wedge z=0$

$P_y: x=0 \wedge z=0$

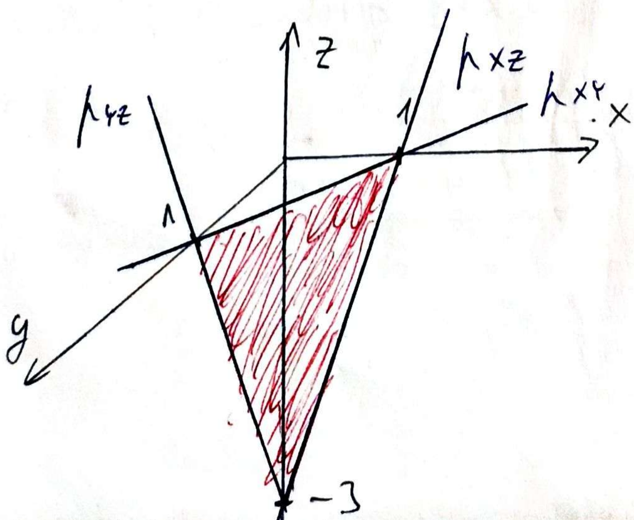
$P_z: x=0 \wedge y=0$

$$P_x: \left.\begin{aligned} 0 &= 1-2k+l \Rightarrow l = 2k-1 \qquad \Rightarrow l=0 \\ 0 &= 6-12k-6l \\ &\Rightarrow 0 = 6-12k-12k+6 \Rightarrow 24k = 12 \Rightarrow k = \tfrac{1}{2} \end{aligned}\right\} P_x[1;0;0]$$

$$P_y: \left.\begin{aligned} 0 &= 2-2k-3l \\ 0 &= 6-12k-6l \Rightarrow l = 1-2k \qquad \Rightarrow l = \tfrac{1}{2} \\ &\Rightarrow 0 = 2-2k-3+6k \Rightarrow 1 = 4k \Rightarrow k = \tfrac{1}{4} \end{aligned}\right\} P_y[0;1;0]$$

$$P_z: \left.\begin{aligned} 0 &= 2-2k-3l \\ 0 &= 1-2k+l \Rightarrow l = 2k-1 \qquad \Rightarrow l = \tfrac{5}{4}-1 = \tfrac{1}{4} \\ &\Rightarrow 0 = 2-2k-6k+3 \Rightarrow 8k = 5 \Rightarrow k = \tfrac{5}{8} \end{aligned}\right\} P_z[0;0;-3]$$

$$z = 6 - \frac{12\cdot 5}{4\cdot 2} - \frac{6}{4} = 6 - \frac{15}{2} - \frac{3}{2} = -3$$



• $\rho = \{[1+t+s;\ 2+3t-s;\ 5t+s];\ t, s \in \mathbb{R}\} \wedge P_x[2;0;0],\ P_y[0;4;0],\ P_z[0;0;-4]$

$\Rightarrow$ <u>priesečnice so súradnicovými rovinami</u>

• $\underline{p_{XY} = \rho \cap \leftrightarrow XY}$

$\Rightarrow p_{XY} = \leftrightarrow P_xP_y : X = P_x + t \cdot \overrightarrow{P_xP_y} \quad \rightarrow \overrightarrow{P_xP_y} = (-2;4;0)$

$\Rightarrow \underline{\underline{p_{XY} = \{[2-2t;\ 4t;\ 0];\ t \in \mathbb{R}\}}}$

• $\underline{p_{XZ} = \rho \cap \leftrightarrow XZ} \quad \rightarrow \overrightarrow{P_xP_z} = (-2;0;-4)$

$\Rightarrow \underline{\underline{p_{XZ} = \{[2-2t;\ 0;\ -4t];\ t \in \mathbb{R}\}}}$

• $\underline{p_{YZ} = \rho \cap \leftrightarrow YZ} \quad \rightarrow \overrightarrow{P_yP_z} = (0;-4;-4)$

$\Rightarrow \underline{\underline{p_{YZ} = \{[0;\ 4-4t;\ -4t];\ t \in \mathbb{R}\}}}$

$\Rightarrow$ <u>prevod na obecnú rovnicu</u>

a) <u>vylúčenie parametra</u>

$$\left.\begin{aligned} x &= 1+t+s \\ y &= 2+3t-s \\ z &= 5t+s \end{aligned}\right\} \quad \left.\begin{aligned} x+y &= 3+4t \\ z+y &= 2+8t \end{aligned}\right\} \oplus \quad \Rightarrow -2x-2y = -6-8t$$

$$-2x-2y+z+y = -6+2$$

$$\rho: \underline{\underline{2x+y-z-4=0}}$$

b) <u>normálový vektor</u>

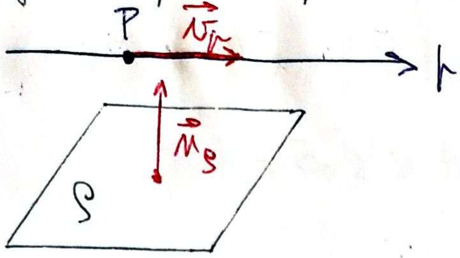
$$\vec{n} = \vec{u} \times \vec{v} \Rightarrow \left.\begin{aligned} &\vec{u}(1;3;5)\ 1;3 \\ &\vec{v}(1;-1;1)\ 1;-1 \end{aligned}\right\} \left.\begin{aligned} n_1 &= 3+5 = 8 \\ n_2 &= 5-1 = 4 \\ n_3 &= -1-3 = -4 \end{aligned}\right\} \vec{n}(2;1;-1)$$

$$\Rightarrow \rho: 2x+y-z+d=0$$

$$P_x: 4+d=0$$

$$\Rightarrow \underline{\underline{\rho: 2x+y-z-4=0}}$$

Vzájemná poloha přímky a roviny



$$\vec{s}_p \cdot \vec{n}_\rho = 0 \iff \vec{s}_p \perp \vec{n}_\rho \iff p \parallel \rho$$

$\Rightarrow p \subset \rho \iff \vec{s}_p \cdot \vec{n}_\rho = 0 \wedge P \in \rho$

$\Rightarrow p \parallel \rho \iff \vec{s}_p \cdot \vec{n}_\rho = 0 \wedge P \notin \rho$

$\Rightarrow p \nparallel \rho \iff \vec{s}_p \cdot \vec{n}_\rho \neq 0$

příklady

- $p = \{[2+t; 3+2t; 1-t]; t \in \mathbb{R}\}$, $\rho: x-2y+z-5=0$

$\vec{s}_p(1;2;-1) \wedge \vec{n}_\rho(1;-2;1) \Rightarrow \vec{s}_p \cdot \vec{n}_\rho = 1-4-1 = -4 \neq 0 \Rightarrow p \nparallel \rho$

$\Rightarrow R = p \cap \rho$: $x-2y+z-5=0$

$$\left.\begin{aligned} x &= 2+t \\ y &= 3+2t \\ z &= 1-t \end{aligned}\right\} \left.\begin{aligned} &2+t-6-4t+1-t-5=0 \\ &-8-4t=0 \Rightarrow t=-2 \end{aligned}\right\} R[0;-1;3]$$

- urči hodnoty parametrů $a, b \in \mathbb{R}$ aby:

1, $p \subset \rho$
2, $p \parallel \rho$
3, $p \nparallel \rho$

$p = \{[a-t; 1+bt; 2-2t]; t \in \mathbb{R}\} \Rightarrow \vec{s}_p = (-1; b; -2)$

$\rho: x+2y-z-10=0 \Rightarrow \vec{n}_\rho = (1;2;-1)$

1, $\vec{s}_p \cdot \vec{n}_\rho = 0 \Rightarrow -1+2b+2=0 \Rightarrow 2b=-1 \Rightarrow b = -\frac{1}{2}$

$P[a;1;2] \in \rho: a+2-2-10=0 \Rightarrow a=10$

2, $\vec{s}_p \cdot \vec{n}_\rho = 0 \Rightarrow b = -\frac{1}{2}$

$P \notin \rho \Rightarrow a \neq 10 \Rightarrow a \in \mathbb{R} \setminus \{10\}$

3, $\vec{s}_p \cdot \vec{n}_\rho \neq 0 \Rightarrow b \neq -\frac{1}{2} \Rightarrow b \in \mathbb{R} \setminus \{-\frac{1}{2}\}$

$a \in \mathbb{R}$

→ Vzájomná poloha dvoch rovín

- $\rho \equiv \sigma \iff \vec{n}_\rho = k \cdot \vec{n}_\sigma \wedge A \in \rho \Rightarrow A \in \sigma$
- $\rho \parallel \sigma \iff \vec{n}_\rho = k \cdot n_\sigma \wedge A \in \rho \Rightarrow A \notin \sigma$
- $\rho \nparallel \sigma \iff \vec{n}_\rho \neq k \cdot \vec{n}_\sigma$

⇒ priesečnica rovín : $p = \rho \cap \sigma$

$$\left.\begin{array}{l} p \subset \rho \Rightarrow \vec{v}_p \perp \vec{n}_\rho \\ p \subset \sigma \Rightarrow \vec{v}_p \perp n_\sigma \end{array}\right\} \vec{v}_p = \vec{n}_\rho \times \vec{n}_\sigma$$

→ 2 možnosti

a) riešim sústavu vyjadrení oboch rovín

⇒ nekonečne mnoho riešení ⇒ body P, Q

$\Rightarrow p: X = P + t \cdot \vec{PQ}$

b) $\vec{v}_p = \vec{n}_\rho \times \vec{n}_\sigma$ ∧ ~~zo~~ sústavy získam bod P

$\Rightarrow p: X = P + t \cdot \vec{v}_p$

→ príklad

- $\rho: x - y + 2z - 4 = 0 \wedge \sigma = \{[2 + t - 2s; 3 + 2t - s; -1 - t + 2s]; t, s \in \mathbb{R}\}$

$$\sigma: \left.\begin{array}{l} \vec{u}(1; 2; -1)\, 1; 2 \\ \vec{v}(-2; -1; 2)\, -2; -1 \end{array}\right\} \vec{n}_\sigma = (3; 0; 3) \rightsquigarrow \left.\begin{array}{l}(1; 0; 1) \\ \vec{n}_\rho(1; -1; 2)\end{array}\right\} \rho \nparallel \sigma$$

$\Rightarrow p = \rho \cap \sigma: \quad \rho: x - y + 2z - 4 = 0$

$$\sigma: \left.\begin{array}{l} x = 2 + t - 2s \\ y = 3 + 2t - s \\ z = -1 - t + 2s \end{array}\right\} \begin{array}{l} 2 + t - 2s - 3 - 2t + s - 2 - 2t + 4s - 4 = 0 \\ -7 - 3t + 3s = 0 \Rightarrow s = t + \frac{7}{3} \end{array}$$

$$\Rightarrow \left.\begin{array}{l} t = 0 \\ s = \frac{7}{3} \end{array}\right\} P = \left[2 - \frac{14}{3}; 3 - \frac{7}{3}; -1 + \frac{14}{3}\right] = \left[-\frac{8}{3}; \frac{2}{3}; \frac{11}{3}\right]$$

$$\Rightarrow \left.\begin{array}{l} \vec{n}_\sigma(1; 0; 1)\, 1; 0 \\ \vec{n}_\rho(1; -1; 2)\, 1; -1 \end{array}\right\} \vec{v}_p = (1; -1; -1)$$

$\Rightarrow p = \{[-\frac{8}{3} + t; \frac{2}{3} - t; \frac{11}{3} - t]; t \in \mathbb{R}\}$

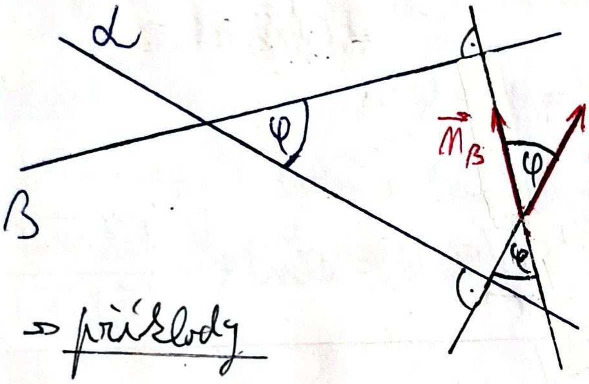
→ Vzdálenost dvou rovnoběžných rovin

$$A \in \alpha \Rightarrow v(\alpha;\beta) = v(A;\beta)$$

$$A[x_0;y_0;z_0] \wedge \beta: ax+by+cz+d=0$$

$$\Rightarrow v(\alpha;\beta) = v(A;\beta) = \frac{|a\cdot x_0 + b\cdot y_0 + c\cdot z_0 + d|}{\sqrt{a^2+b^2+c^2}}$$

→ Odchylka dvou různoběžných rovin



⇒ chci menší úhel ⇒ $\varphi \in (0;90\rangle$

abs. hodn.

$$\Rightarrow \cos(\varphi) = \frac{|\vec{n}_\alpha \cdot \vec{n}_\beta|}{|\vec{n}_\alpha|\cdot|\vec{n}_\beta|}$$

⇒ příklady

- $\rho: 3x-5y+2z-10=0 \Rightarrow \vec{n}_\rho(3;-5;2)$
  $\sigma: -6x+10y-4z-12=0 \Rightarrow \vec{n}_\sigma(-6;10;-4)$
  $\vec{n}_\rho = -\frac{1}{2}\vec{n}_\sigma \Rightarrow \rho \parallel \sigma$

  $3x-5y+2z+6=0$

  $v(\rho;\sigma): A\in\rho: x,y=0 \Rightarrow 2z-10=0 \Rightarrow z=5 \Rightarrow A[0;0;5]$

  $$v(A;\sigma) = \frac{|3\cdot 0 - 5\cdot 0 + 2\cdot 5 + 6|}{\sqrt{9+25+4}} = \frac{16}{\sqrt{38}} \doteq 2{,}6$$

- $\rho: 3x-5y+2z-10=0 \Rightarrow \vec{n}_\rho(3;-5;2)$
  $\sigma: x+2y-z-2=0 \Rightarrow \vec{n}_\sigma(1;2;-1)$
  $\vec{n}_\rho \neq k\cdot\vec{n}_\sigma \Rightarrow \rho \nparallel \sigma$

⇒ průsečnice

$$\left.\begin{aligned} \rho&: 3x-5y+2z-10=0 \\ \sigma&: 2x+4y-2z-4=0 \end{aligned}\right\} \oplus\ 5x-y-14=0 \Rightarrow y=5x-14$$

$$\Rightarrow \sigma: x+2(5x-14)-z-2=0 \Rightarrow x+10x-28-z-2=0 \Rightarrow z=11x-30$$

$$\left.\begin{aligned} \Rightarrow x&=x \\ y&=5x-14 \\ z&=11x-30 \end{aligned}\right\} p=\{[t;-14+5t;-30+11t];\ t\in\mathbb{R}\}$$

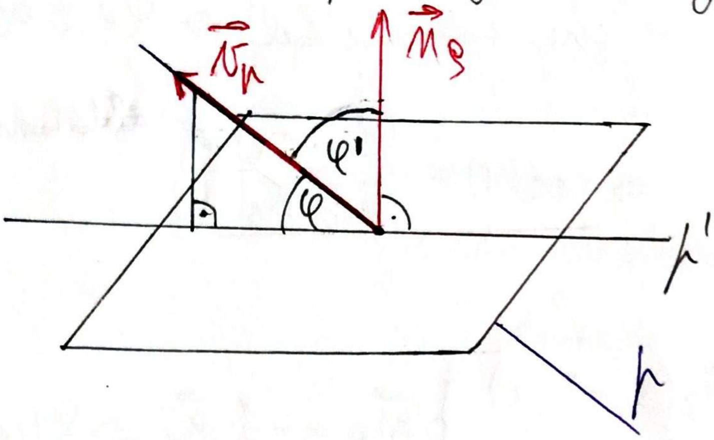
→ vzdálenost přímky od roviny - $p \parallel \rho$

$P \in p \Rightarrow v(p;\rho) = \vec{v}(P;\rho)$

$\Rightarrow P[x_0;y_0;z_0] \wedge \rho: ax+by+cz+d=0$

$$\Rightarrow v(p;\rho) = \frac{|a\cdot x_0 + b\cdot y_0 + c\cdot z_0 + d|}{\sqrt{a^2+b^2+c^2}}$$

→ odchylka přímky od roviny - $p \nparallel \rho$



$\varphi' = 90 - \varphi$

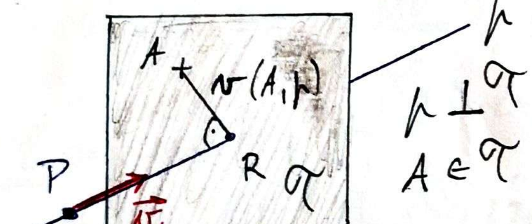
$$\cos(90-\varphi) = \sin(\varphi) = \frac{|\vec{v}_p \cdot \vec{n}_\rho|}{|\vec{v}_p|\cdot|\vec{n}_\rho|}$$

→ odchylka dvou přímek - $p \nparallel q \vee p \times q$ - stejně jako v π_2

$$\Rightarrow \cos(\varphi) = \frac{|\vec{v}_p \cdot \vec{v}_q|}{|\vec{v}_p|\cdot|\vec{v}_q|}$$

→ vzdálenost dvou přímek - $p \parallel q$ = vzdálenost bodu od přímky - A, p

a) stereometricky



$p \perp \sigma$

$A \in \sigma$

$A[-3;1;2] \wedge p = \{[-1;2+t;2t];\ t \in \mathbb{R}\}$

$\sigma: A, \vec{n}_\sigma = \vec{v}_p = (-1;1;2)$

$\left.\begin{array}{l}\sigma: -x+y+2z+d=0 \\ A: 3+1+4+d=0\end{array}\right\} \underline{\underline{\sigma: -x+y+2z-8=0}}$

$\Rightarrow R = p \cap \sigma: t+2+t+4t-8=0 \Rightarrow 6t=6 \Rightarrow \underline{\underline{t=1}} \Rightarrow R[-1;3;2]$

$\Rightarrow v(A,p) = |AR| = \sqrt{4+4+0} = \underline{\underline{2\sqrt{2}}}$

b) analyticky → $R \in p \Rightarrow R = [-1;2+t;2t];\ t = ?$

$\Rightarrow \vec{AR} = (3-t;1+t;-2+2t) \rightarrow \vec{AR} \perp \vec{v}_p \Rightarrow \vec{AR}\cdot\vec{v}_p = 0$

$\Rightarrow -3+t+1+t-4+4t=0 \Rightarrow -6+6t=0 \Rightarrow \underline{\underline{t=1}}$

$\Rightarrow \vec{AR} = (2;2;0) \Rightarrow |\vec{AR}| = \sqrt{4+4} = \underline{\underline{2\sqrt{2}}}$