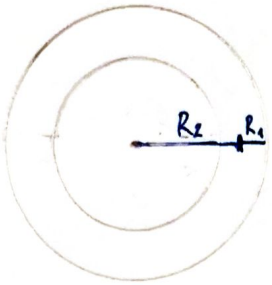
## • Moment setrvačnosti mezikruží - $J_{\odot}$



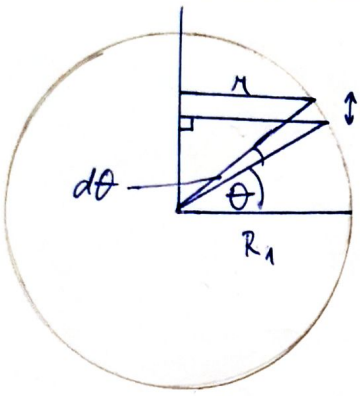

$$J_{\odot} = \int_{R_2-R_1}^{R_2+R_1} J_{\odot} = \int_{R_2-R_1}^{R_2+R_1} r^2\,dm = 2\pi\sigma \int_{R_2-R_1}^{R_2+R_1} r^3 dr = \frac{1}{2}M\,\frac{(R_2+R_1)^4-(R_2-R_1)^4}{(R_2+R_1)^2-(R_2-R_1)^2}$$

$$dm = \sigma\,dS = \sigma\,2\pi r\,dr =$$

$$\sigma = \frac{M}{V} = \frac{M}{\pi\left[(R_2+R_1)^2-(R_2-R_1)^2\right]}$$

$$\Rightarrow \underline{\underline{J_{\odot} = \frac{1}{2}M\left[(R_2+R_1)^2+(R_2-R_1)^2\right]}}$$

## • Moment setrvačnosti toru - $J_{\bullet}$




$$J_{\bullet} = 2\cdot\int_0^{\frac{\pi}{2}} J_{\odot} = \int_0^{\frac{\pi}{2}} \left[(R_2+r)^2+(R_2-r)^2\right] dm$$

$$dm = \rho\,dV = \rho\pi\left[(R_2+r)^2-(R_2-r)^2\right] dh$$

$$r = R_1\cos\theta \quad , \quad dh = r\,d\theta$$

$$dh = R_1\left(\sin(\theta+d\theta)-\sin(\theta)\right) = R_1\,\frac{\sin(\theta+d\theta)-\sin\theta}{d\theta}\,d\theta = R_1\cos\theta\,d\theta$$

$$\Rightarrow J_{\bullet} = \rho\pi\int_0^{\frac{\pi}{2}}\left[(R_2+r)^4-(R_2-r)^4\right] r\,d\theta = \rho\pi\int_0^{\frac{\pi}{2}}\left[8R_2^3 r+8R_2 r^3\right] r\,d\theta =$$

$$(R_2-r)^4 = R_2^4-4R_2^3 r+6R_2^2 r^2-4R_2 r^3+r^4$$

$$= \rho\pi\int_0^{\frac{\pi}{2}}\left[8R_2^3R_1^2\cos^2\theta+8R_2R_1^4\cos^4\theta\right] d\theta = \rho\pi\left[R_2^3R_1^2\,8\int_0^{\frac{\pi}{2}}\cos^2\theta\,d\theta + R_2R_1^4\,8\int_0^{\frac{\pi}{2}}\cos^4\theta\,d\theta\right]$$

$$* \quad 8\int_0^{\frac{\pi}{2}}\cos^2\theta\,d\theta = 4\int_0^{\frac{\pi}{2}}\left(1+\cos(2\theta)\right) d\theta = 4\left[\theta+\frac{1}{2}\sin(2\theta)\right]_0^{\frac{\pi}{2}} = 4\cdot\frac{\pi}{2} = \underline{2\pi}$$

$$* \quad 8\int_0^{\frac{\pi}{2}}\cos^4\theta\,d\theta = 2\int_0^{\frac{\pi}{2}}\left(1+\cos(2\theta)\right)^2 d\theta = 2\int_0^{\frac{\pi}{2}}\left(1+2\cos(2\theta)+\cos^2(2\theta)\right) d\theta = 2\int_0^{\frac{\pi}{2}}\left(1+2\cos(2\theta)+\frac{1}{2}\left(1+\cos(4\theta)\right)\right) d\theta =$$

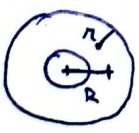
$$= 2\left[\frac{3}{2}\theta+\sin(2\theta)+\frac{1}{8}\sin(4\theta)\right]_0^{\frac{\pi}{2}} = 2\cdot\frac{3}{2}\cdot\frac{\pi}{2} = \underline{\underline{\frac{3\pi}{2}}}$$

$$\Rightarrow J_{\bullet} = \rho\pi\left(2\pi R_2^3R_1^2+\frac{3}{2}\pi R_2R_1^4\right) = \qquad \left| \rho = \frac{M}{V} = \frac{M}{\pi R_1^2\cdot 2\pi R_2} = \frac{M}{2\pi^2R_2R_1^2}\right.$$

$$= \frac{1}{2}M\cdot\frac{1}{\pi R_2R_1^2}\left(2\pi R_2^3R_1^2+\frac{3}{2}\pi R_2R_1^4\right)$$

$$= \frac{1}{2}M\cdot\left(2R_2^2+\frac{3}{2}R_1^2\right) = MR_2^2+\frac{3}{4}MR_1^2$$

→ Pro torus




$$\underline{\underline{J_{\bullet} = mR^2+\frac{3}{4}mr^2}}$$